STRESS - STRENGTH INTERFERENCE

PROBABILITY OF FAILURE

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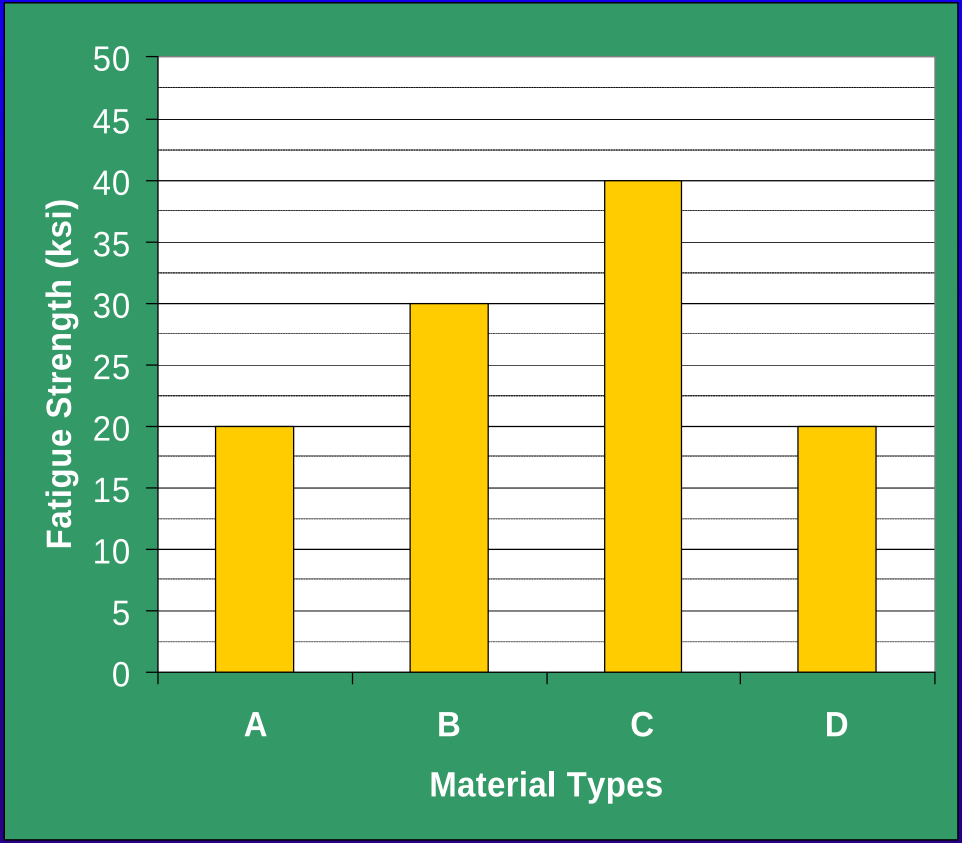
QUIZ 1!

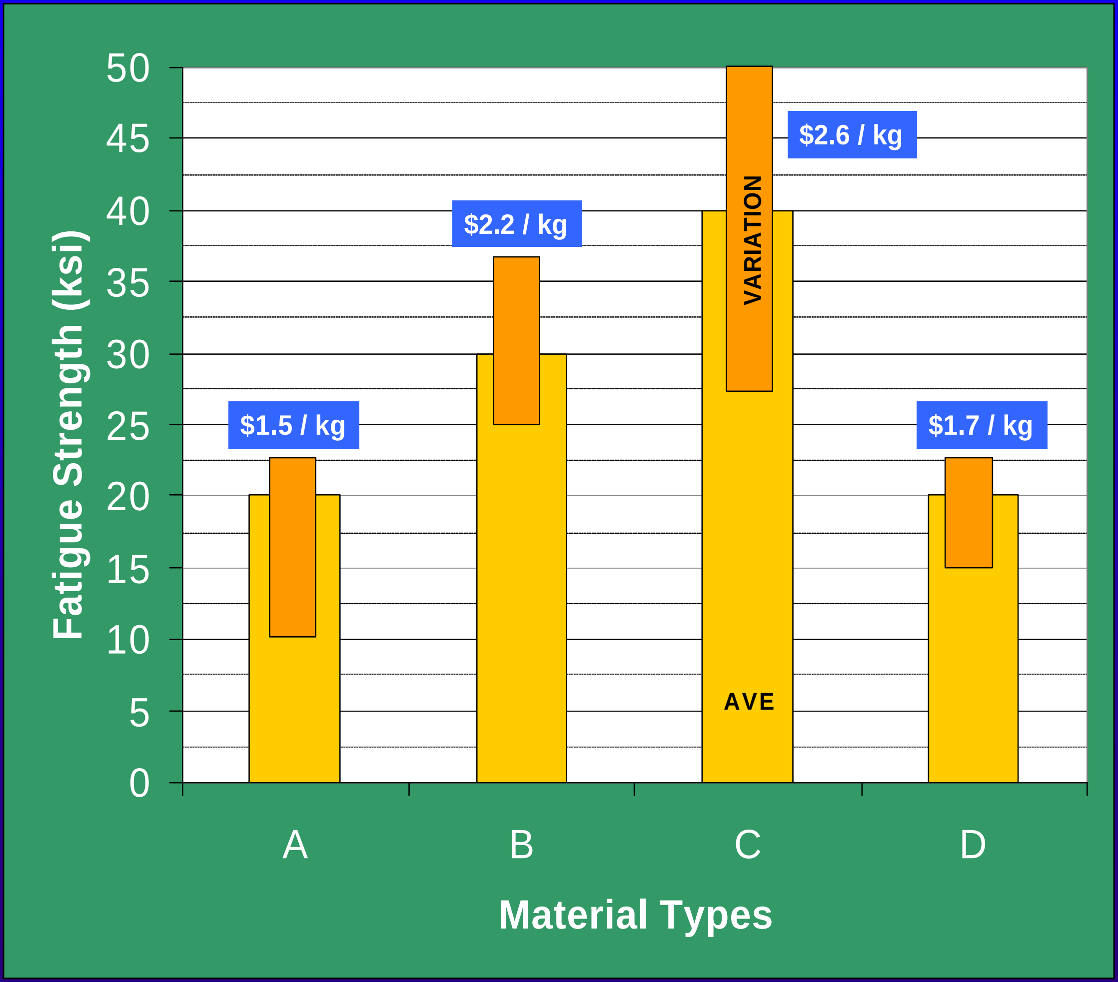
$$\text{DESIGN MARGIN} = \frac{\text{Strength}}{\text{Design Stress}}$$

- **Which one of the materials would you choose for a higher fatigue strength?**
- **Would you pay a premium for material "D" over material "A"?**



VARIATION IS THE *ENEMY*…

- **Would the additional information like the variation in strength change your answers?**
- **Is material "C" better then "B"?**
- **How would you quantify the difference between these materials?**
- **Is design margin alone a good criteria to select a material?**



Early Practice - Design Margin

- Early practice in stress-strength relationship dealt almost entirely along the lines of design margin. Factor of safety!
- Design margin approach use the mean value of stress & strength ignoring the natural scatter that each may possess.
- Utilization of design margin is justified when
  - It is based on considerable experience
  - Component design changes are not too different than the existing design.
    - Geometry, processing, function

Recent Practice – Probability of Failure

- The variation in stress and strength results in a statistical distribution and a natural scatter in these variables.
- When these two distributions interfere, that is when stress becomes higher than strength, failure results.
- Means of expressing these distributions in a practical engineering sense and means of calculating the resulting interference (probability of failure) is the heart of this seminar.

Outline

- Definition of failure - Unreliability
- Reliability in simple terms
- Part Strength & Stress
- Normal Distribution
- Probability of failure
- Reliability quantified
- Example

Definition of Failure - Unreliability

- **Failure**
  - The inability to meet customer required function
- **Failure Mode**
  - The manner in which the item fails, not the display of the failure
    - It is very important to identify the root cause and separate the failure modes
- **Mission Disabling Failure**
  - When mission is interrupted such that the item cannot or should not be operated until repair occurs

How Do Customers Talk About Reliability

- " A system that does what I want (function), when and where I want to use it (conditions), for as long as I want to use it (time) "
- " No surprises - no unscheduled downtime"
- " Get me up and running quickly when failures occur "
  - This is as important as not having a failure in the first place.

Definition of Reliability

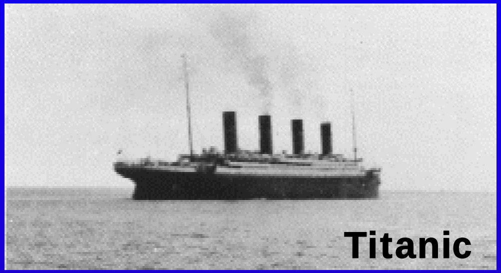
- "The ability of an item to perform a required function under stated conditions for a stated period of time"
- "Quality over time"
- "It is also defined and/or measured as the probability that an item performs…"
  - It is with this definition that we can quantify Reliability.

Reliability Measures

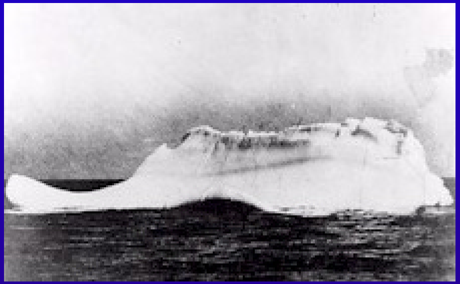
- **Cumulative failure rate at a stated time**
  - **Repairs Per Hundred (RPH) within warranty period**
- **Instantaneous failure rate or hazard rate**
  - **Failure rate per hour, month, mile in service**
- **Time it takes to fail**
  - **Mean Time Between Failures (MTBF)**
  - **B_{10} life – time at which 10% of the items have failed**
- **Probability of failure**

Causes of Different Failure Types

- **Infant Mortality**
  - **Manufacturing & assembly issues**
  - **Quality control issues**
  - **Supplier Issues**

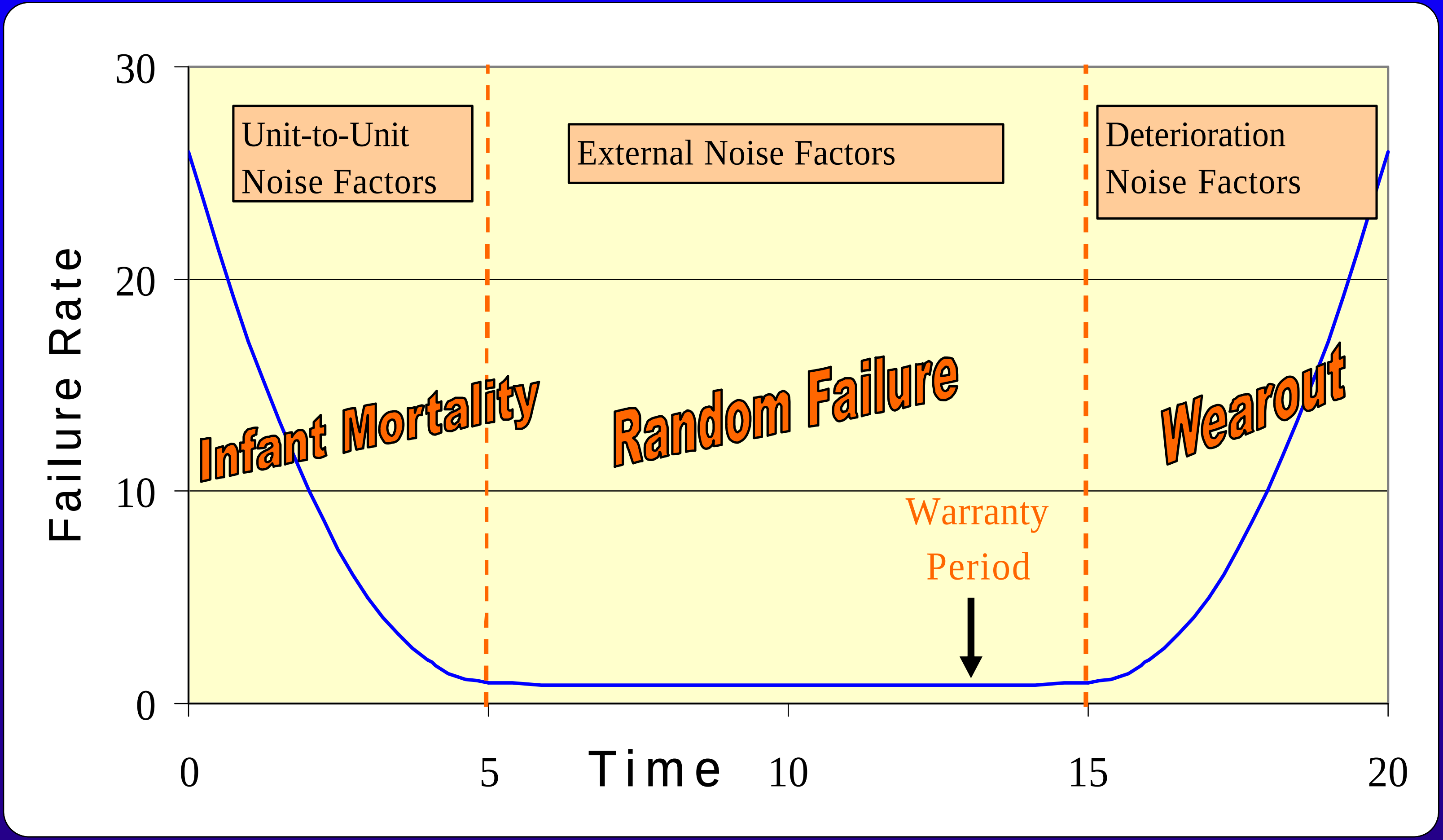


- **Random Failures**
  - **Interference of inherent strength and experienced stress during operation**
  - **Misapplication and/or abuse**



- **Wear Out Failures**
  - **Fatigue, wear and part deterioration**
  - **Preventive maintenance issues**
  - **Service issues**

Reliability Bathtub Curve



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Stress Acting on a Part

- **The operating stress imposed on the part is random.**
- **Stress acting on a part changes with**
  - **Time**
    - **Climbing a hill with full, part & no load**
    - **City vs. high way driving**
  - **Ambient conditions (temperature, pressure)**
  - **Part to part – variation in geometry**
  - **User to user**
    - **18 year old driving dad's Porsche**

Part Strength

- A given part has certain physical properties which, if exceeded, will cause failure.
- Part strength is a random variable that can be represented by a statistical distribution.
- A parts strength varies from
  - Lot to lot – Difference in chemical composition
  - Manufacturer to manufacturer – Process
  - Ambient conditions
    - Change in material properties with temperature and humidity
    - At low temperatures parts may shrink and reduce sealing pressure (Space shuttle failure)

Stress & Strength Distribution

- Random variation of stress and strength can be expressed with different distributions
  - Normal, Log Normal, Exponential, Weibull
- Both stress and strength can be represented with any combination of the above distributions.
  - Normal – Normal, Normal – Weibull,
Log Normal – Exponential, Weibull – Weibull
- For the purpose of this seminar we will assume a normal distribution for both stress and strength
  - Math behind normal-normal distribution is simpler

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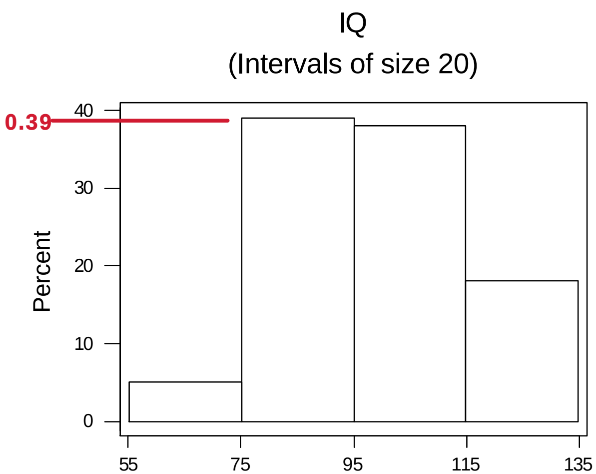
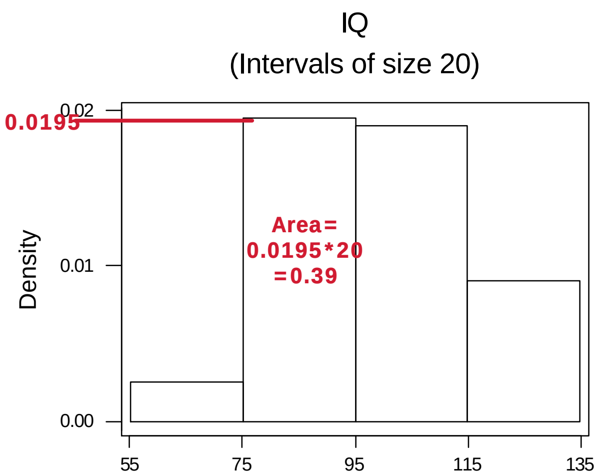
- **Definition of failure - Unreliability**
- **Reliability in simple terms**
- **Part Strength & Stress**
- **Normal Distribution**
  - **Probability Density Function**
- **Probability of failure**
- **Reliability quantified**
- **Example**

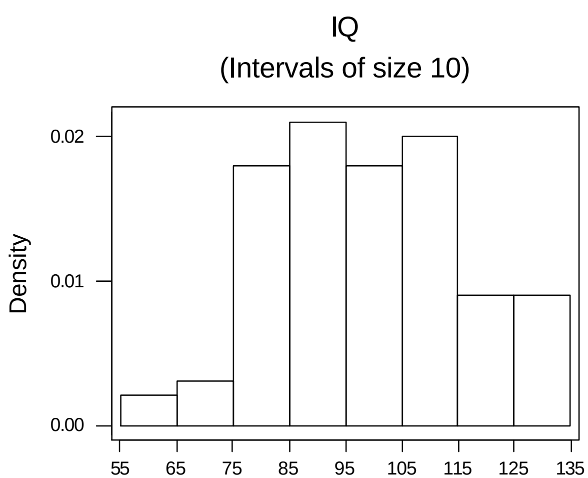
Normal Distribution



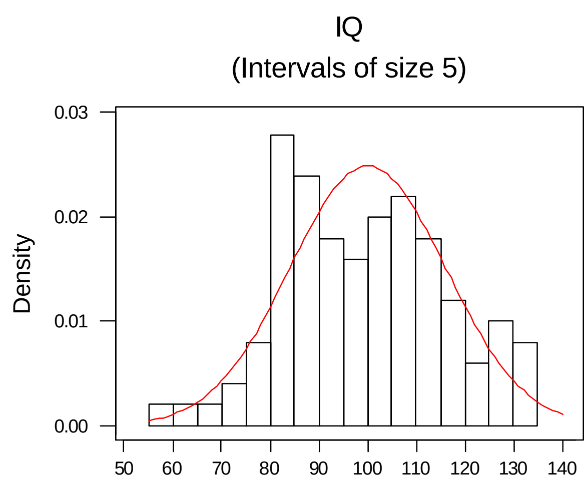
Percent Histogram



Probability = Area of Rectangle



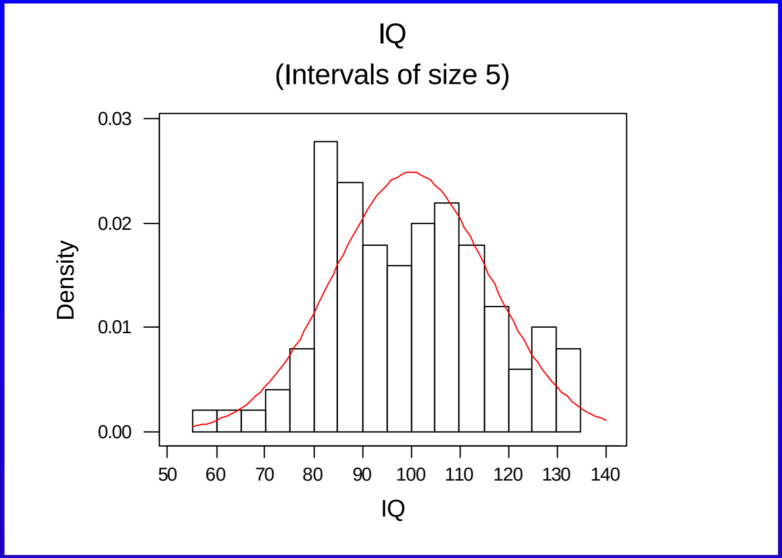
Decrease interval size



Decrease interval size more

Normal Distribution Characteristics

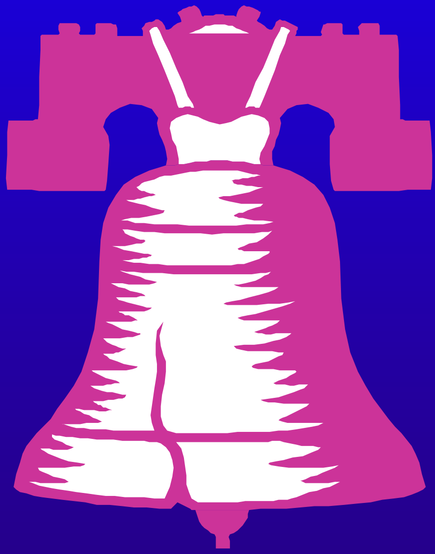
- **Symmetric, bell-shaped curve.**
- **Shape of curve depends on population** mean μ **and** standard deviation σ.
- **Center of distribution is μ.**

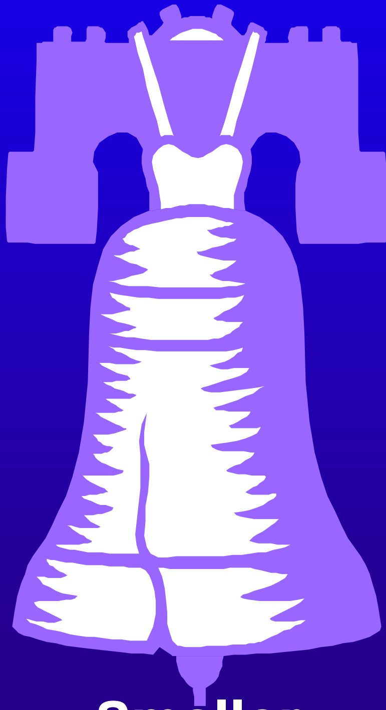


- **Spread is determined by standard deviation σ.**
- **Most values fall around the mean, but some values are smaller and some are larger.**

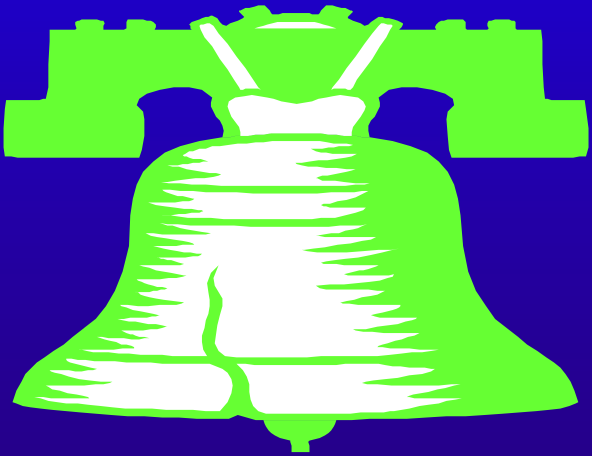
Normal Distribution:
Effect of Mean & Standard Deviation

- **The mean and standard deviation affect the shape of the normal distribution**





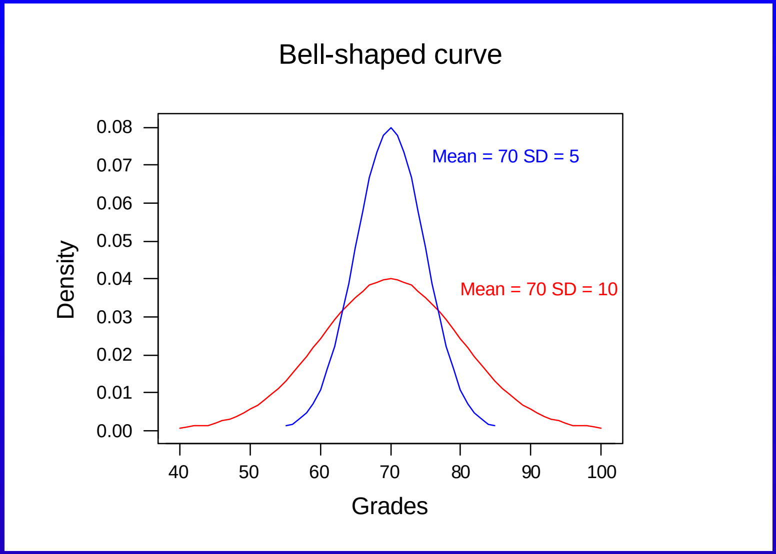
Smaller standard deviation

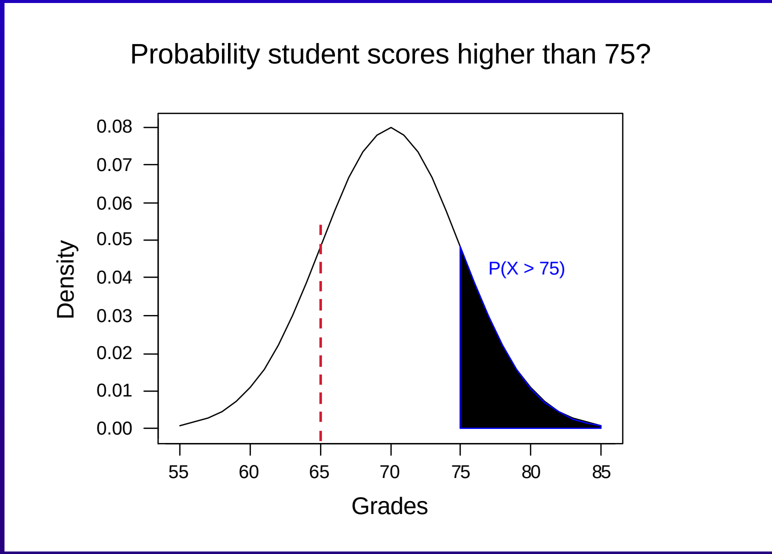


Larger standard deviation

Probability Density Function (PDF)

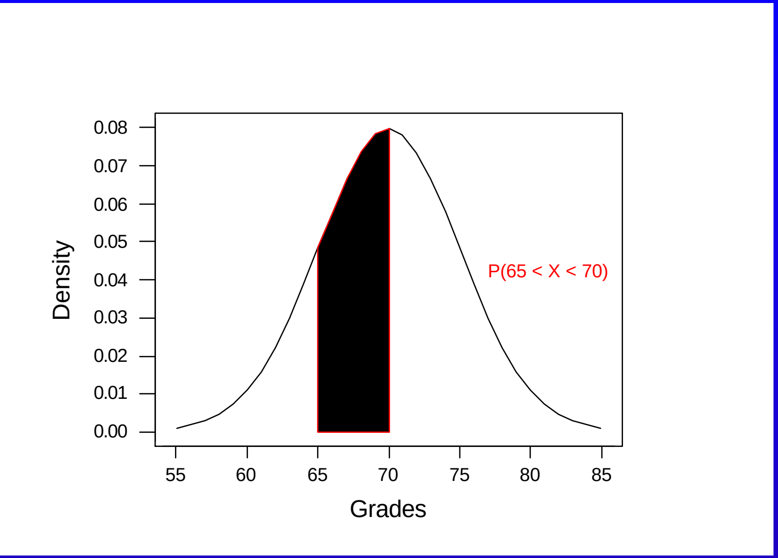
- **The curve describes probability of getting any range of values**
  - **P(X > 120), P(X>75), P(65 > X > 75)**
- **Probability is the area under the curve**
- **Area under the whole curve is 1**
- **Probability of getting specific number is 0**
  - **P(X=120) = 0**



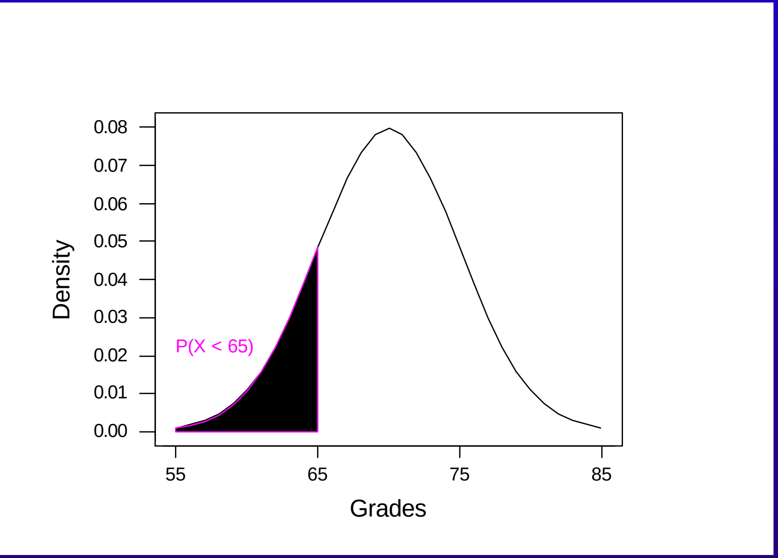


Probability = Area under curve

- **Probability of all grades falling between 65 & 70.**
  - **P (65 < X < 70)**



- **Probability of all grades falling below 65**
  - **Is always a function of the instructor!**
  - **Has nothing to do with how much you study!**

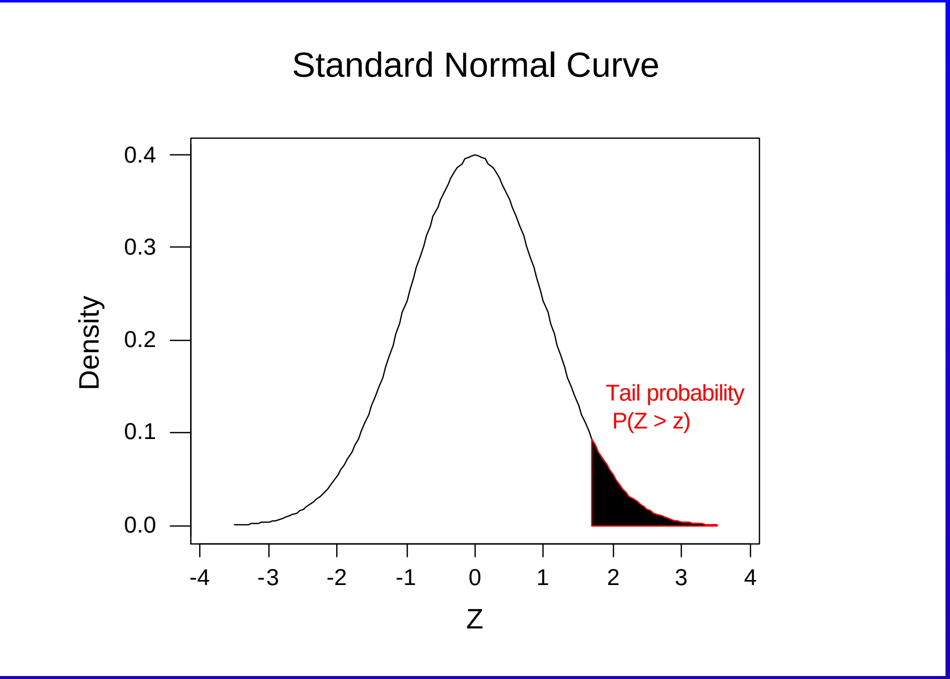


Probability = Area under curve

- Integral? Calculus?! I am kidding, right?
- But somebody did all the hard work for us!
- We just need a table of probabilities for every possible normal distribution.
- But there are an infinite number of normal distributions (one for each m and s)!!
- Solution is to "standardize".

Standard Normal Curve

- **Take a normally distributed value X**
- **Subtract its mean m from it**
- **Divide by its standard deviation s.**
- **Call the resulting value Z.**



$$Z = \frac{(X - m)}{s}$$

- **Z is called the** standard normal**. Its mean m is 0 and standard deviation s is 1.**
- **Probability of failure, Unreliability is calculated from standardized normal distribution as Failure=P(Z).**
- **Reliability = (1 – Unreliability)**

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Stress & Strength Interference

- **X (Strength) and Y (Stress) are normally distributed with mean values μ_x and μ_y and variances s^2_x and s^2_y**
- **Define I = X – Y → (Strength – Stress)**
  - **Mean value $\mu_I = \mu_x - \mu_y$**
  - **Variance $s^2_I = s^2_x + s^2_y$**
- **Normalize function I=(Strength-Stress) so that standard statistical tables can be used**

$$Z = \frac{(I - m_I)}{s_I} = \frac{I - (m_X - m_Y)}{s^2_x + s^2_y}$$

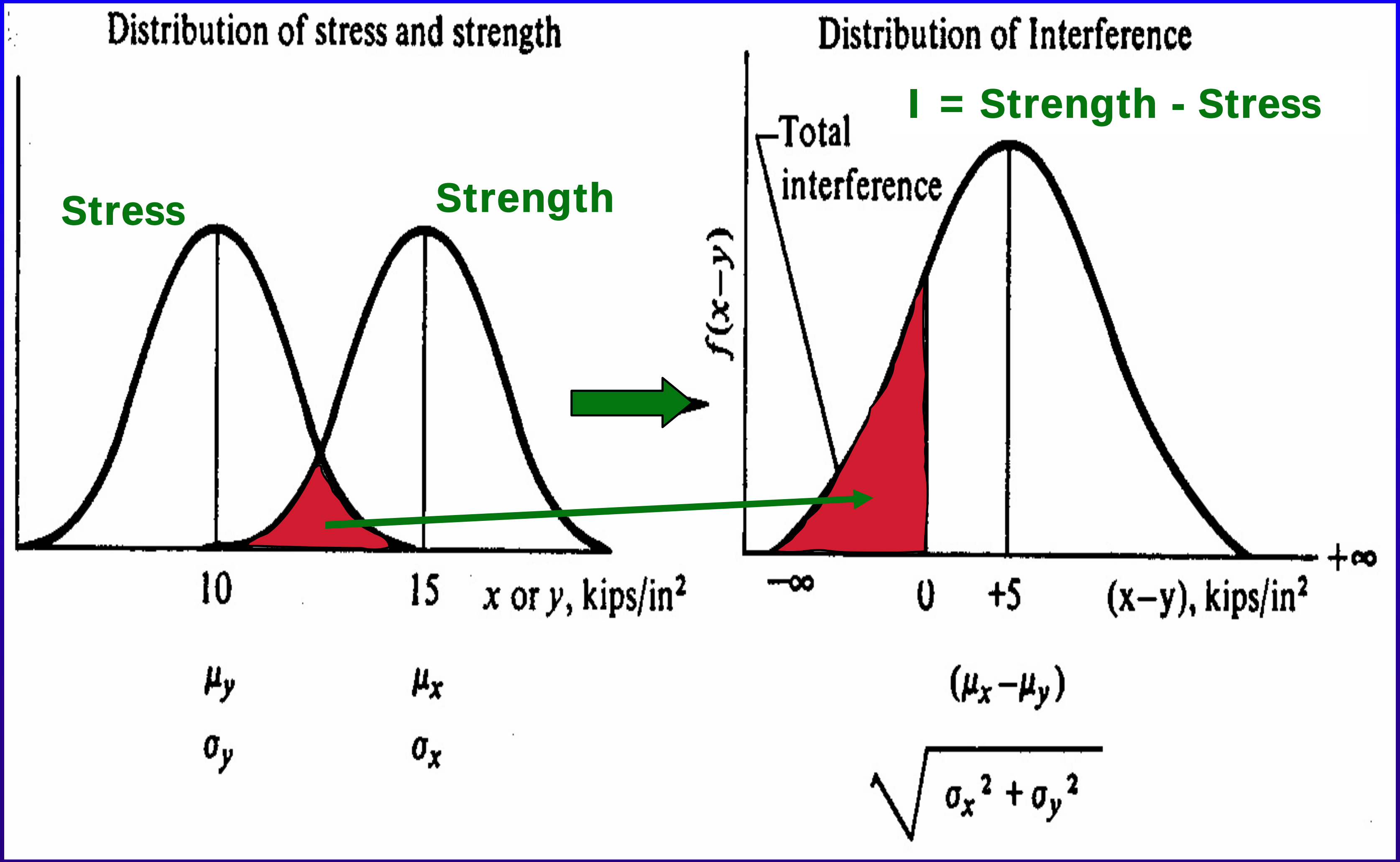
Stress & Strength Interference

- **Part stress must be equal or exceed part strength for failure to occur**
  - **Stress Y >= Strength X**
  - **I (X – Y) =< 0**
- **Area under the normalize function where I = (Strength-Stress) = 0 is consequently the probability of failure**

$$\textbf{Unreliability} = Z = \frac{0 - (m_X - m_Y)}{s^2_x + s^2_y}$$

Interference of Two Normal Distributions

Part stress must exceed strength for failure to occur



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Example - Probability of Failure

- A component has a strength which is normally distributed with a mean value of 5000 N and standard deviation of 400 N. The load it has to withstand is also normally distributed with a mean and standard deviation 3500 N and 600 N. What is the reliability of this component under the given load application?

$$Z = \frac{0-(5000-3500)}{\sqrt{400^2+600^2}}$$

$$Z = 2.08$$

$$Unreliability = 0.0188$$

$$Reliability = 1-0.0188$$

$$= 0.9812$$

z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139

- Probability of failure is 1.88 out of 100

Example – Repairs Per Hundred

- Unreliability = 0.0188 & Reliability = 0.9812
- RPH (Repairs Per Hundred) = 100 * Unreliability

  = 100 * 0.0188

  RPH = 1.9
  - 1.9 parts out of 100 will fail
  - 19 parts out of 1000 will fail
- RPH will be 0.4 if the load standard deviation is reduced to 400N from 600N.
- Design Margin for both cases is $\frac{5000}{3500} = 1.42$

Example – Cost of Robustness

- If the repair cost of such a failure is $750 and annual engine build rate is 40,000, How much premium can the manufacturer pay for reduced standard deviation - robustness?

$$\text{Number of Failures} = \frac{(1.88 - 0.4)}{100} \times 40{,}000 = 592 \text{ eng/year}$$

$$\text{Cost per Engine} = \frac{592 \frac{\text{engines}}{\text{year}} * 750 \frac{\$}{\text{engine}}}{40{,}000 \frac{\text{engines}}{\text{year}}} = 11.10 \frac{\$}{engine}$$

- **Up to $11.1 per engine** can be paid to reduce the load variability (standard deviation) from 600N to 400N
  - Larger crank damper to reduce torsional amplitudes
- **What is the price for 592 happy customers?**