ECE 302 Solution to Homework Assignment 4

Note: To obtain credit for an answer, you must provide adequate justification. Also, if it is possible to obtain a numeric answer, you should not only find an expression for the answer, but also solve for the numeric answer.

1. Suppose that the random variable \( N \) has PMF

\[
P_N(n) = \begin{cases} 
  cn^2 & n \in \{1, 2, 3\} \\
  0 & \text{otherwise.}
\end{cases}
\]

(a) What is the value of \( c \)?

Solution: The random variable \( N \) is a discrete random variable having sample space \( S_N = \{1, 2, 3\} \) so by Theorem 2.1(b) (which relies on the second axiom of probability) applies. The theorem states that the sum of the probabilities over the sample space is 1. Thus

\[
1 = \sum_{n \in S_N} P_N(n) = c(1)^2 + c(2)^2 + c(3)^2 = 14c
\]

and we conclude that \( c = 1/14 \) and

\[
P_N(n) = \begin{cases} 
  n^2/14 & n \in \{1, 2, 3\} \\
  0 & \text{otherwise.}
\end{cases}
\]

(b) Find the cumulative distribution function \( F_N(n) \).

Solution: Applying the definition of the cumulative distribution, (D2.11, p. 62),

\[
F_N(n) = P[N < n] = \begin{cases} 
  0 & n < 1 \\
  1/14 & 1 \leq n < 2 \\
  5/14 & 2 \leq n < 3 \\
  1 & 3 \leq n
\end{cases}
\]

(c) Find the expected value \( E[N] \).

Solution: Applying the definition of the expected value, (D2.14, p. 66),

\[
E[N] = \sum_{n \in S_N} nP_N(n) = 1(1/14) + 2(4/14) + 3(9/14) = 36/14 = 18/7
\]

(d) Find the variance \( \text{Var}[N] \).

Solution: Applying Theorem 2.13, p. 78, the variance is

\[
\text{Var}[N] = E[N^2] - (E[X])^2
\]
so we need the value of expected value of \( N^2 \). Applying Theorem 2.10, p. 74, with \( g(N) = N^2 \), we obtain

\[
E[N^2] = \sum_{n \in S_N} N^2 P_N(n) = 1^2(1/14) + 2^2(4/14) + 3^2(9/14) = 98/14 = 7,
\]

thus

\[
\text{Var}[N] = 7 - \left( \frac{18}{7} \right)^2 = \frac{19}{49}.
\]

(e) Find a mode of \( N \) and indicate whether or not it is unique. If not, describe the set of other values that are also modes.

**Solution:** By definition (D2.12, p. 66), the mode of \( N \) is the value \( n \in S_N \) having largest probability. Thus the mode is 3.

(f) Find a median of \( N \) and indicate whether or not it is unique. If not, describe the set of other values that are also medians.

**Solution:** According to the textbook’s definition of median, D2.13, p. 66, the median \( x_{med} \) of a random variable \( X \) must satisfy

\[
P[X < x_{med}] = P[X > x_{med}].
\]

This definition has the unfortunate property that many distributions, including the one under discussion, do not have a median. Specifically, let \( n^* < 3 \), then \( P[N > n^*] \geq P[N = 3] = 9/14 \) while \( P[N < n^*] \leq P[N \leq 2] = 5/14 < 9/14 \) so we conclude that \( n_{med} \) must be at least 3. If we let \( n^* \geq 3 \), then \( P[X > n^*] = 0 \), while \( P[N < n^*] = 1 \) so \( n_{med} \) cannot be at least three. Having covered all of the possibilities, we must conclude that \( n_{med} \) does not exist.

OTOH, others define the median of the random variable \( X \) to be the value \( x_{med} \) that satisfies

\[
P[X \leq x_{med}] \geq 1/2 \quad \text{and} \quad P[X \geq x_{med}] \geq 1/2.
\]

Now we have that

\[
P[N \leq 3] = 1 \geq 1/2 \\
P[N \geq 3] = 9/14 \geq 1/2
\]

so the median according to this definition is \( n_{med} = 3 \).

If we want to know the median to know roughly where the distribution is “centered”, then we see that this is a better definition because it applies to more distributions and seems to yield the required information. Moral: consider what information you want before blindly using a formula in a book.
2. Suppose that the random variable \(N\) has PMF
\[
P_N(n) = \begin{cases} 
  cn^2 & n \in \{-2, -1, 1, 2\} \\
  0 & \text{otherwise}
\end{cases}
\]
(a) What is the value of \(c\)?

**Solution:** The random variable \(N\) is a discrete random variable having sample space \(S_N = \{-2, -1, 1, 2\}\) so by Theorem 2.1(b) (which relies on the second axiom of probability) applies. The theorem states that the sum of the probabilities over the sample space is 1. Thus
\[
1 = \sum_{n \in S_N} P_N(n) = c(-2)^2 + c(-1)^2 + c(1)^2 + c(2)^2 = 10c
\]
and we conclude that \(c = 1/10\) and
\[
P_N(n) = \begin{cases} 
  n^2/10 & n \in \{-2, -1, 1, 2\} \\
  0 & \text{otherwise}
\end{cases}
\]
(b) Find the cumulative distribution function \(F_N(n)\).

**Solution:** Applying the definition of the cumulative distribution, (D2.11, p. 62),
\[
F_N(n) = P[N < n] = \begin{cases} 
  0 & n < -2 \\
  4/10 & -2 \leq n < -1 \\
  5/10 & -1 \leq n < 1 \\
  6/10 & 1 \leq n < 2 \\
  1 & n \geq 2
\end{cases}
\]
(c) Find the expected value \(E[N]\).

**Solution:** Applying the definition of the expected value, (D2.14, p. 66),
\[
E[N] = \sum_{n \in S_N} nP_N(n) = -2(4/10) + -1(1/10) + 1(1/10) + 2(4/10) = 0
\]
(d) Find the variance \(\text{Var}[N]\).

**Solution:** Applying Theorem 2.13, p. 78, the variance is
\[
\text{Var}[N] = E[N^2] - (E[X])^2
\]
so we need the value of expected value of \(N^2\). Applying Theorem 2.10, p. 74, with \(g(N) = N^2\), we obtain
\[
E[N^2] = \sum_{n \in S_N} N^2P_N(n) = (-2)^2(4/10) + (-1)^2(1/10) + 1^2(1/10) + 2^2(4/10)
\]
\[
= 34/10 = 17/5,
\]
thus
\[
\text{Var}[N] = \frac{17}{5} - 0^2 = \frac{17}{5}.
\]
(e) Find a mode of \( N \) and indicate whether or not it is unique. If not, describe the set of other values that are also modes.

**Solution:** By definition (D2.12, p. 66), the mode of \( N \) is the value or values \( n \in S_N \) having largest probability, thus 2 and -2 are modes of \( N \).

(f) Find a median of \( N \) and indicate whether or not it is unique. If not, describe the set of other values that are also medians.

**Solution:** For any value \( n^* \) in the open interval \((-1, 1)\), \( P[N < n^*] = P[N > n^*] \) so every value in the open interval is a median of \( N \). In this case, the book’s definition sufficed. Note that each of these medians also satisfies the improved definition mentioned in the solution to the previous problem.
3. Suppose that on any given February day in Indianapolis it rains with probability 5/7 or snows with probability 2/7 independently of what happens on any other day. Let \( N \) be the number of snowy days in a week. If you use Matlab, you must include two things in the material you turn in: (1) if you used a .m file, a printout of the .m file that you, personally, wrote, and (2) a transcript of the part of your Matlab session in which your results are obtained.

(a) Find the PMF of \( N \). (First give the expression in terms of the appropriate values, then evaluate the expression to obtain numerical values.)

**Solution:** The expression for the PMF of \( N \) is as follows:

\[
P_N(n) = \binom{7}{n} \left(\frac{2}{7}\right)^n \left(\frac{5}{7}\right)^{7-n}.
\]

The numerical values for \( P_N(n) \) are calculated by the script shown in Figure 1 and are found in Figure 2.

(b) Find the CDF of \( N \) using the numerical values from part (a).

**Solution:** See Figure 1 for the Matlab script used to calculate the CDF and Figure 2 for the CDF \( F_Nn \).

(c) Find the expected value of \( N \) (using the numerical values).

**Solution:** See Figure 1 for the Matlab script used to calculate the expected value, which, according to the output showing in Figure 2 is 2. (Which is what we would expect.)

(d) Find the variance of \( N \) (using the numerical values).

**Solution:** See Figure 1 for the Matlab script used to calculate the variance, which, according to the output showing in Figure 2 is approximately 1.4286.

(e) Find the expected number of rainy days in the week (using the numerical values).

**Solution:** See Figure 1 for the Matlab script used to calculate the expected value, which, according to the output showing in Figure 2 is 5. (Which is what we would expect.)

(f) Find the variance of the number of rainy days in the week (using the numerical values).

**Solution:** See Figure 1 for the Matlab script used to calculate the variance, which, according to the output showing in Figure 2 is approximately 1.4286.
Figure 1: Matlab Script that calculates answers to Problem 3, parts (a) through (f)

```matlab
PNn = zeros(8,1);
for index = 0:7,
    choose(index+1) = factorial(7)/(factorial(index)*factorial(7-index));
    PNn(index+1) = choose(index+1)*((2/7)^index)*((5/7)^(7-index));
end;
PNn'

FNn = zeros(8,1);
for index = 0:7,
    FNn(index+1) = sum(PNn(1:index+1));
end;
FNn'

EN = sum([0:7]'.*PNn)
VN = sum([0:7]'*[0:7]'*PNn)-EN^2

PRr = zeros(8,1);
for index = 0:7,
    choose(index+1) = factorial(7)/(factorial(index)*factorial(7-index));
    PRr(index+1) = choose(index+1)*((2/7)^(7-index))*((5/7)^index);
end;
PRr'

ER = sum([0:7]'.*PRr) %%% = 7-ER
VR = sum([0:7]'*[0:7]'*PRr)-ER^2
```
Figure 2: Answers to Problem 3, parts (a) through (f)

$$\text{>> hw4p3}$$

$$\text{PNn'} =$$

0.0949 0.2656 0.3187 0.2125 0.0850 0.0204 0.0027 0.0002

$$\text{FNn'} =$$

0.0949 0.3605 0.6792 0.8917 0.9767 0.9971 0.9998 1.0000

$$\text{EN} =$$

2

$$\text{VN} =$$

1.4286

$$\text{PRr'} =$$

0.0002 0.0027 0.0204 0.0850 0.2125 0.3187 0.2656 0.0949

$$\text{ER} =$$

5

$$\text{VR} =$$

1.4286
4. Now, continuing the previous problem, suppose that when it rains, the probability of receiving less than 1 inch of rain is 1/8, the probability of obtaining at least 1 but less than 3 inches is 1/2, the probability of obtaining at least 3 but less than 5 inches is 1/4, and the probability of obtaining 5 inches or more is 1/8. Suppose further that when it snows the probability of receiving less than 1 inch is 7/8 and the probability of obtaining an inch or more is 1/8.

(a) Define appropriate events and random variables and give the probabilities of the events and the PMF’s of the random variables. Include the sample space, any event space you have defined, as well as the ranges of the random variables.

Solution: Define the event $R_i$ to be the event that it rains on the $i$th day of the week. Then $P[R_i] = 5/7$ $\forall i \in \{1, 2, 3, 4, 5, 6, 7\}$. The sample space is then $S = \{s_1, s_2, s_3, s_4, s_5, s_6 : s_i \in \{R_i, R_i^c\}\}$.

(b) Find the conditional probability of the amount of precipitation received when it rains.

Solution: We are given the conditional PMF of the amount of precipitation received when it rains, namely, for each $R_i$,

$$P_{D|R_i}(d) = \begin{cases} 
1/8 & 0 < d < 1 \\
1/2 & 1 \leq d < 3 \\
1/4 & 3 \leq d < 5 \\
1/8 & 5 \leq d \\
0 & \text{otherwise}
\end{cases}$$

(c) Find the conditional probability of the amount of precipitation received when it snows.

Solution: We are given the conditional PMF of the amount of precipitation received when it snows, namely, for each $R_i^c$,

$$P_{D|R_i^c}(d) = \begin{cases} 
7/8 & 0 < d < 1 \\
1/8 & 1 \leq d \\
0 & \text{otherwise}
\end{cases}$$

continued...
(d) Find the expected value of the amount of rain obtained on a rainy day.

Solution: Technically, the problem does not give enough information to determine this since we don't have any particular reason to assume that the probability is distributed uniformly across each range. However, to get a rough estimate, we could use the midpoint of each interval for the finite intervals and one larger than the left end of the interval for the infinite intervals. This would then yield

\[ E[D|R_i] = \sum_{j=1}^{4} d_j P_{D|R_i}(d_j) \]

\[ = (1/2)(1/8) + 2(1/2) + 4(1/4) + 6(1/8) \]

\[ = 45/16 \approx 2.8125 \]

as an estimate for the amount of precipitation received on a rainy day.

(e) Find the expected value of the amount of snow obtained on a snowy day.

Solution: Technically, the problem does not give enough information to determine this since we don’t have any particular reason to assume that the probability is distributed uniformly across each range. However, to get a rough estimate, we could use the midpoint of each interval for the finite intervals and one larger than the left end of the interval for the infinite intervals. This would then yield

\[ E[D|R_c] = d_1 P_{D|R_c}(d_1) + d_2 P_{D|R_c}(d_2) = (1/2)(7/8) + 2(1/8) = 11/16 \]

as an estimate for the amount of precipitation received on a snowy day.

(f) Find the expected amount of precipitation on any arbitrary day.

Solution: As before, technically we don’t have enough information here, but using the approximations above, we obtain

\[ E[D] = E[D|R_i]P[R_i] + E[D|R_c]P[R_c] \]

\[ = (45/16)(5/7) + (11/16)(2/7) \]

\[ = 247/112 \approx 2.2054 \].
5. Consider an error correcting binary code, which uses 8 bits to represent each of the values 0 through 7. The code words and the values they represent are

<table>
<thead>
<tr>
<th>Code Word</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>00000000</td>
<td>0</td>
</tr>
<tr>
<td>00001111</td>
<td>1</td>
</tr>
<tr>
<td>00110011</td>
<td>2</td>
</tr>
<tr>
<td>01010101</td>
<td>3</td>
</tr>
<tr>
<td>10101010</td>
<td>4</td>
</tr>
<tr>
<td>11001100</td>
<td>5</td>
</tr>
<tr>
<td>11100000</td>
<td>6</td>
</tr>
<tr>
<td>11111111</td>
<td>7</td>
</tr>
</tbody>
</table>

Using this code, one bit-error per word can be corrected.\(^1\)

Suppose that the probability of an error in an individual bit is 1/50 independent of the correctness of any other bit.

(a) Find the PMF of the random variable \(N\) representing the number of bit errors in a received word and then find \(E[N]\).

**Solution:** We model the receipt of a code word as a set of \(m = 8\) Bernoulli trials in which each of the \(m\) bits is wrong with probability \(p = 1/50\). Thus \(N\) is a Binomial \((m, p)\) variable.

\[
P_N(n) = \binom{m}{n} \left( \frac{1}{50} \right)^n \left( \frac{49}{50} \right)^{m-n}
\]

and by Theorem 2.7(a), p. 69, the expected value of \(N\) is

\[
E[N] = \sum_{n=0}^{8} nP_N(n) = mp = 8(1/50) = 4/25.
\]

(b) Let \(B\) be the event that the received word can be correctly decoded. What is the probability of \(B\)?

**Solution:** The probability \(q\) that the word can be correctly decoded is the probability that there is not more than one bit error, so

\[
P[B] = P[\{N = 0\} \cup \{N = 1\}] = P_N(0) + P_N(1)
\]

\[
= \left( \frac{49}{50} \right)^8 + \binom{8}{1} \left( \frac{49}{50} \right)^7 \left( \frac{1}{50} \right)
\]

\[
= \left( \frac{49}{50} \right)^7 \left( \frac{49}{50} + \frac{8}{50} \right) = \left( \frac{49^7(57)}{50^8} \right) = q \approx .8681
\]

---

\(^1\)You can see this by noting that each code word differs from any other code word in 4 bits. Thus if one bit is in error, the word received is still closer to the code word sent than it is to any other code word.
(c) Suppose that you need to send a message consisting of an ordered string of three words. What is the probability that the entire three-word message will be correctly decoded?

Solution: The probability that all three words are correctly decoded is equal to the product of the probabilities that each individual word is correctly decoded. Thus the probability is $q^3$ or approximately 0.6543.

(d) Let $N$ be the number of words of the three word message that are correctly decoded. Find the PMF of $N$.

Solution: The receipt of the three sequential code words can be modelled as three independent Bernoulli trials with probability $q$ so $P_N(n)$ is a Binomial $(3, q)$ random variable. Thus

$$P_N(n) = \begin{cases} \binom{3}{0} q^0 (1-q)^3 & n = 0 \\ \binom{3}{1} q^1 (1-q)^2 & n = 1 \\ \binom{3}{2} q^2 (1-q)^1 & n = 2 \\ \binom{3}{3} q^3 (1-q)^0 & n = 3 \\ 0 & \text{otherwise} \end{cases}$$

and the probability of receiving zero words correctly is approximately 0.0413, of one is 0.2346, of two is 0.4440, and of three is 0.2801. (Check: 0.0413 + 0.2346 + 0.4440 + 0.2801 = 1.)

(e) Repeat part (a) for the case where the probability of an error in an individual bit is $1/5$ independent of the correctness of any other bit.

Solution: We model the receipt of a code word as a set of $m = 8$ Bernoulli trials in which each of the $m$ bits is wrong with probability $p = 1/5$. Thus $N$ is a Binomial $(m, p)$ variable.

$$P_N(n) = \binom{m}{n} \left(\frac{1}{5}\right)^n \left(\frac{4}{5}\right)^{m-n}$$

and by Theorem 2.7(a), p. 69, the expected value of $N$ is

$$E[N] = \sum_{n=0}^{8} n P_N(n) = mp = 8(1/5) = 1.6.$$ 

Since the code corrects only one error and the expected number of errors per code word received is 1.6, we don’t expect to get much useful information from this communication.

(f) Repeat part (b) for the case where the probability of an error in an individual bit is $1/5$ independent of the correctness of any other bit.
Solution: The probability \( q \) that the word can be correctly decoded is the probability that there is not more than one bit error, so

\[
P[B] = P[N = 0] + P[N = 1]
\]

\[
= \left( \frac{4}{50} \right)^8 + \left( \frac{8}{1} \right) \left( \frac{4}{5} \right)^7 \left( \frac{1}{5} \right) = \left( \frac{4}{5} \right)^7 \left( \frac{4}{5} + \frac{8}{5} \right)
\]

\[
= \left( \frac{47}{5^8} \right) = q \approx 0.5033.
\]

Thus the probability of decoding a word correctly is barely more than 1/2, and from the calculations in the previous parts, it is clear that the probability of receiving three consecutive code words correctly is approximately 1/8. Since we could guess at random with probability 1/8 of success, we see that while the code is helpful if the probability of error is low, if the probability of error is too high, it negates the benefit of the code.
6. A student takes three courses in the current semester. In the previous semester, the student took the prerequisite for each of these courses and earned an A with probability 0.1, a B with probability 0.5, or a C with probability 0.4, independently of the grade in any other course. You don’t know what the student got in the previous semester.

Now, if the student received an A in the prerequisite, then you know that this semester, the student will receive an A with probability 1/3, a B with probability 1/2, or a C with probability 1/6. OTOH if the student received a B in the prerequisite, there is a 1/6 probability of the student getting an A, a 1/3 probability of getting a B, a 1/3 probability of getting a C, and a 1/6 probability of getting a D. Finally, if the student received a C in the prerequisite, the student receives a B with probability 1/6, a C with probability 1/3, a D with probability 1/3 and an F with probability 1/6.

An A is worth 4 points, a B 3, and a C 2 in calculating the student’s GPA.

(a) What is the PMF of the student’s GPA in the previous semester?

**Solution:** In the previous semester, the possible outcomes are given in Table 1, which was generated using the Matlab script in Figure 3. Please note that the Matlab scripts given in these solutions are not optimized for efficiency, but rather for readability.

<table>
<thead>
<tr>
<th>Possible Grades</th>
<th>Number of Ways to Order</th>
<th>Probability</th>
<th>GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>1</td>
<td>0.0640</td>
<td>12/3 = 4</td>
</tr>
<tr>
<td>AAB</td>
<td>3</td>
<td>0.2400</td>
<td>11/3</td>
</tr>
<tr>
<td>ABB</td>
<td>3</td>
<td>0.3000</td>
<td>10/3</td>
</tr>
<tr>
<td>BBB</td>
<td>1</td>
<td>0.1250</td>
<td>9/3 = 3</td>
</tr>
<tr>
<td>BBC</td>
<td>3</td>
<td>0.0480</td>
<td>8/3</td>
</tr>
<tr>
<td>BCC</td>
<td>3</td>
<td>0.1200</td>
<td>7/3</td>
</tr>
<tr>
<td>CCC</td>
<td>1</td>
<td>0.0750</td>
<td>6/3 = 2</td>
</tr>
<tr>
<td>CCA</td>
<td>3</td>
<td>0.0120</td>
<td>8/3</td>
</tr>
<tr>
<td>CAA</td>
<td>3</td>
<td>0.0150</td>
<td>10/3</td>
</tr>
<tr>
<td>ABC</td>
<td>6</td>
<td>0.0010</td>
<td>9/3 = 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>27</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

(b) What is the expected value of the student’s GPA in the previous semester?

**Solution:** The expected GPA calculated by the Matlab script in Figure 3 is 2.7.
Figure 3: Matlab script for computing the PMF of the previous semester GPA.

```matlab
%%% Problem 6(a),6(b)

count = 0;
P = zeros(10,1);
num = zeros(10,3);
for index1 = 0:3; %%% number of A's
    for index2 = 0:3; %%% number of B's
        for index3 = 0:3; %%% number of C's
            index_sum = index1 + index2 + index3; %%% "number" of grades
            if index_sum == 3 ; %%% number of grades must be 3
                count = count + 1;
                if max([index1,index2,index3]) == 3;
                    P(count) = (0.1^index1)*(0.5^index2)*(0.4^index3);
                elseif max([index1,index2,index3]) == 2;
                    P(count) = 3*(0.1^index1)*(0.5^index2)*(0.4^index3);
                elseif max([index1,index2,index3]) == 1;
                    P(count) = 6*(0.1^index1)*(0.5^index2)*(0.4^index3);
                end;
                GPA(count) = (4*index1 + 3*index2 + 2*index3)/3;
                num(count,:) = [index1,index2,index3];
            end;
        end;
    end;
end;
sum(P)
num
P
GPA
eGPA = GPA'*P
```
(c) What is the PMF of the student’s GPA in the current semester?

Solution: Let $G_1$ represent the first-semester grade in a prerequisite, and $G_2$ represent a second-semester grade, i.e. the grade in the follow-on course. The problem statement gives us that

$$P[G_2|G_1 = A] = \begin{cases} 
  1/3 & G_2 = A \\
  1/2 & G_2 = B \\
  1/6 & G_2 = C 
\end{cases} \quad (1)$$

$$P[G_2|G_1 = B] = \begin{cases} 
  1/6 & G_2 = A \\
  1/3 & G_2 = B \\
  1/3 & G_2 = C \\
  1/6 & G_2 = D 
\end{cases} \quad (2)$$

$$P[G_2|G_1 = C] = \begin{cases} 
  1/6 & G_2 = B \\
  1/3 & G_2 = C \\
  1/3 & G_2 = D \\
  1/6 & G_2 = F 
\end{cases} \quad (3)$$

in addition to the probabilities $P[G_1 = A] = 1/10, P[G_2 = B] = 1/2$, and $P[G_1 = C] = 2/5$. We can then find the probability, e.g. that $P[G_2 = A]$ using the Law of Total Probability. Namely,


Note that the expressions for $P[G_2 = A]$ and $P[G_2 = D]$ have two terms whereas the expressions for $P[G_2 = B]$ and $P[G_2 = C]$ have three terms and the expression for $P[G_2 = F]$ has only one term.

Now, we saw in the problem on HW4 that there were 10 different combinations of grades that could be obtained in the first semester. However, in this semester, there will be more because each grade can be $A$, $B$, $C$, $D$, or $F$. After running the Matlab script, we see that there are 35 different combinations of grades.

Obviously, this will be a job for Matlab. I will encode the conditional probabilities in a table. Notice that the table includes the zero probability cases so that I can conveniently script the computation. The Matlab script is shown in Figure 4.

<table>
<thead>
<tr>
<th>$G_1$</th>
<th>$G_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$B_1$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1/3</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1/2</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1/6</td>
</tr>
<tr>
<td>$D_2$</td>
<td>0</td>
</tr>
<tr>
<td>$F_2$</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 4: Matlab script for computing the PMF of the current semester GPA.

%%% Problem 6(c),6(d)

%%% First find probabilities for G2.

PG2G1 = [1/3 1/6 0; 1/2 1/3 1/6; 1/6 1/3 1/3; 0 1/6 1/3; 0 0 1/6];
PG1 = [1/10 1/2 2/5];
PG2 = zeros(5,1);

for G2index = 1:5; %%% A2, B2, C2, D2, F2
    for G1index = 1:3; %%% A1, B1, C1
        PG2(G2index)= PG2(G2index)+PG2G1(G2index,G1index)*PG1(G1index);
    end;
end;

count = 0;
mc = zeros(35,1);
P = zeros(35,1);
GPA = zeros(35,1);
um = zeros(35,5);

for index1 = 0:3; %%% number of A's
    for index2 = 0:3; %%% number of B’s
        for index3 = 0:3; %%% number of C’s
            for index4 = 0:3; %%% number of D’s
                for index5 = 0:3; %%% number of F’s
                    index_sum = index1 + index2 + index3 + index4 + index5;
                    if index_sum == 3 ; %%% number of grades must be 3
                        count = count + 1;
                        mc(count) = factorial(3);
                        mc(count) = (((((mc(count)/factorial(index1))/...factorial(index2))/factorial(index3))/...factorial(index4))/factorial(index5));
                        P(count) = mc(count)*((PG2(1))ˆindex1)*...((PG2(2))ˆindex2)*((PG2(3))ˆindex3)*...((PG2(4))ˆindex4)*((PG2(5))ˆindex5);
                        GPA(count) = (4*index1 + 3*index2 + 2*index3 + 1*index4)/3;
                        num(count,:) = [index1,index2,index3,index4,index5];
                    end;
                end;
            end;
        end;
    end;
end;

sum(P)
num
P
GPA
eGPA = GPA’*P
(d) What is the expected value of the student’s GPA in the current semester?

Solution: According to the results of the computations performed by the Matlab script, the expected value of the current semester’s GPA is 2.1667.

(e) What is the PMF of the student’s combined GPA?

Solution: The PMF of the combined GPA is given by the variable $P$ for the GPA’s in the order they occur in the variable $GPA$. (Technically we should sum the probabilities that give the same GPA, but that is trivial, so I’ll skip it. – And no, you may not use that justification for skipping steps in your solutions.)

(f) What is the expected value of the student’s combined GPA?

Solution: The expected value of the combined GPA is the sum of the expected GPA’s divided by two, namely 2.4333.