1. As a function of time, the number of emails arriving at a the WINLAB mail server is \( N(t) \), a Poisson stochastic process. In any 20 second interval, the expected number of arrivals is 2.

(a) What is the arrival rate, \( \lambda \), of the process?
(b) Observe the process for 10 seconds. The number of emails arriving in 10 seconds is a random variable, \( M \). What is the PMF of \( M \)?
(c) What is the probability of observing more than one query in the first 20 seconds?

2. The duration of a phone call is an exponential random variable with expected value 3 minutes. All phone calls have independent durations. In a one-week period someone makes 16 calls. The total calling time is a random variable \( T \) (in minutes).

(a) What is the expected duration \( \mu_T \) of all calls combined?
(b) What is the variance of the duration of all calls combined?
(c) Use the central limit theorem to estimate the probability that the duration of all calls combined will be more than one hour.
(d) Use the central limit theorem to estimate the probability that the duration of all calls combined will be between 39 and 51 minutes.

3. If, for the random variable \( Y \) with expected value \( \mu_Y = -4 \), Chebyshev’s inequality states that \( P[-10 < Y < 2] \geq 0.5 \), what is the variance of \( Y \)?

4. \( V \) is a Gaussian random variable with \( \mu_V = 0 \), and \( \text{Var} [V] = 64 \).

(a) For the sample mean of 100 independent samples of \( V \), what is the largest value of \( c \) that satisfies the inequality \( P[|M_{100}(V)| \geq c] \leq 0.05 \)?
(b) For the sample mean of \( n \) independent samples of \( V \), what is the smallest value of \( n \) that satisfies the inequality \( P[M_n(V) < 1] \geq 0.9 \)?

5. The duration of an ordinary (voice) phone call is an exponential random variable with expected value 3 minutes. The duration of an Internet (modem) call is an exponential random variable with expected value 15 minutes. Based on the random variable, \( T \), the duration of one phone call, perform a test to decide whether the call is a voice call \( (H_0) \) or a modem call. \( (H_1) \). The probability of a voice call is 0.6.

(a) If the acceptance region for \( H_0 \) is \( A_0 = \{ T < 10 \ \text{minutes}\} \), what is the probability of a false alarm?
(b) If the acceptance region for \( H_0 \) is \( A_0 = \{ T < 10 \ \text{minutes}\} \), what is the probability of a missed detection?
(c) If the acceptance region for $H_0$ is $A_0 = \{T < 10 \text{ minutes}\}$, what is the probability of an error?

(d) If the acceptance region for $H_0$ is $A_0 = \{T < t^* \text{ minutes}\}$, $P_{MISS} = 0.2$. What is $t^*$?

6. Random variable $W$ is the sum of a signal $V$ and noise $N$. $V$ and $N$ are independent. $V$ has a uniform PDF with limits -1 and +1. $N$ has a uniform PDF with limits -0.5 and +0.5. What is the covariance of $W$ and $V$?

7. Stochastic process $W(t)$ is the sum of a signal process $V(t)$ and noise process $N(t)$. $V(t)$ and $N(t)$ are independent. $V(t)$ is Gaussian with mean $\mu_V(t)$. $N$ is Gaussian with mean $\mu_N(t)$.

(a) What are the auto-covariances and autocorrelations of $V(t)$ and $W(t)$?

(b) Under what conditions is $W(t)$ stationary?

(c) What is the cross-correlation of $V(t)$ and $W(t)$?

(d) Under what conditions are $W(t)$ and $V(t)$ jointly wide-sense stationary?

(e) State the conditions on $W(t)$ and $V(t)$ required for you to be able to compute the power spectral density. Assume that these hold and compute them.

(f) State the conditions on $W(t)$ and $V(t)$ required for you to be able to compute the cross-spectral density. Assume that these hold and compute it.