Final Exam (One Version)

Each student must calculate, document, and submit his or her own answers, corresponding to the individual parameter set assigned by the instructor. Students may discuss the problems with anyone, and may consult any reference books as well as their lecture notes. Students should indicate with whom they have discussed problems and give credit where credit is due.

All steps in obtaining answers must be shown.

1. Consider the linear time-invariant system given by

$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u \tag{1}$$

$$y = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} x. \tag{2}$$

   (a) Compute $\mathcal{V}^*$, the supremal (A, B)-invariant subspace, and find all friends F of $\mathcal{V}^*$.

   (b) Compute $\mathcal{R}^*$, the supremal controllability subspace contained in $\mathcal{V}^*$.

   (c) Given $x_1 = 0$ and $x_2 = [1 \ \ 0 \ \ 0 \ \ 0]^T$, show that for any given finite time T we can find a control $u = Fx + Gv$ that takes x_1 to x_2, in time T, along a straight line.

   (d) If we let $x_1 = [0 \ \ 1 \ \ -1 \ \ 0]^T$ and $x_2 = [0 \ \ k \ \ -k \ \ 0]^T$, for what values of k can x_2 be reached from x_1 in finite time?

2. Consider the linear time-invariant system given by

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + Ew \tag{3}$$

$$y = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} x \tag{4}$$

   where w is a disturbance input.

   (a) Derive the minimum constraint on E such that the Disturbance Decoupling Problem (DDP) is solvable.

   (b) Find a state feedback $u = Fx + v$ that solves the DDP.

   (c) Can we find a $u = Fx + v$ that solves the DDP for any E that meets the minimum constraint obtained above, while stabilizing the closed-loop system?

3. Obtain a balanced realization for the linear time-invariant system given by

$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} u \tag{5}$$

$$y = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} x. \tag{6}$$

4. Find the optimal linear feedback control for the system of Problem 4, assuming that the input weighting matrix is the identity. (Use the output equation $y = Cx$ from Problem 4 to correspond to $z = Dx$ in the text.) Calculate the exact value of the matrix P using Matlab. Then, write scripts to find the matrix P by (a) the Newton's method algorithm, (b) the Matrix sign function method algorithm, and (c) the Schur method algorithm. Plot the value of P(1,1) vs. the iteration for all three methods on the same set of axes. Comment on the plot. (You must submit your scripts in addition to the plots.)