Midterm Exam

Each student must calculate, document, and submit his or her own answers, corresponding to the individual parameter set assigned by the instructor. Students may discuss the problems with anyone, and may consult any reference books as well as their lecture notes. Students should indicate with whom they have discussed problems and give credit where credit is due.

All steps in obtaining answers must be shown.

Consider the matrices

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} & 0 \\ 0 & 0 & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix} \quad \text{and} \quad A_0 = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix} \tag{1}$$

1. (Chapter 0) Examine the maps A and A_0 and for each calculate the following:
   (a) eigenvalues with multiplicities
   (b) eigenvectors and generalized eigenvectors
   (c) characteristic and minimal polynomials
   (d) rational canonical structure
   (e) image and kernel
2. (Chapter 1) Now, let

$$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{2}$$

   (a) Find $\mathscr{B}$, the image of B.
   (b) Find the reachable subspaces for the pairs (A, B) and (A_0, B).
   (c) For the system represented by the pair (A, B) only, find an input $u(s)$ that takes the state $x(0) = [1\ 0\ 1\ 0\ 1]^T$ to $x(1) = 0$.
   (d) For each of A and A_0, use Theorem 1.2 to decompose the state space $\mathscr{X}$ into A-invariant subspaces.
3. (Chapter 2) Consider the system represented by (A, B). Partition the complex plane as $\mathbb{C} = \mathbb{C}_g \cup \mathbb{C}_b$ where $\mathbb{C}_b = \{\lambda : \mathcal{R}e(\lambda) > -2\}$.
   (a) Design a linear feedback to place the poles of the system (A, B) at -1, $-2 \pm j$, $-3 \pm 2j$, or, if this is not possible, explain (show) why.
   (b) Find the "good" part $\alpha_g(\lambda)$ and the "bad" part $\alpha_b(\lambda)$ of the minimal polynomial of A_0.
   (c) Decompose the state space into the "good" and "bad" modal subspaces $\mathscr{X}_g(A_0)$ and $\mathscr{X}_b(A_0)$
   (d) Apply Theorem 2.2 to place as many poles as possible in $\mathbb{C}_g$, the "good" part of the complex plane.

4. (Chapter 3) Let
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

   For each of the two systems (C, A) and (C, A_0) determine the following:

   (a) Is the system observable? If not, what is the unobservable subspace?

   (b) Design a full-order dynamic observer for the observable system.

   (c) For the system with the unobservable subspace, identify the "factor system" as described in Section 3.2 and design a full-order dynamic observer for it.

   (d) For the observable subsystem, design a minimal order dynamic observer.