Some Notes on Morphisms

On page 8 of the textbook\textsuperscript{1}, Wonham writes: “Essential to any grasp of algebra is a command of Greek prefixes.” Let’s look at the prefixes used in the text to describe a map $C : \mathcal{X} \rightarrow \mathcal{Y}$. Recall that $\mathcal{X}$ is called the domain of $C$ and $\mathcal{Y}$ is called the codomain of $C$. Recall also that the kernel of $C$ is the set of all elements of the domain that $C$ maps to zero, and the image of $C$ is the set of all elements of the codomain that can be expressed as $Cx$ for some $x \in \mathcal{X}$.

**auto-** auto means self, as in “autobiography”. $C$ is an automorphism if (i) its codomain is the same as its domain, \textit{i.e.} $\mathcal{Y} = \mathcal{X}$, and (ii) the kernel of the map $C$ is trivial, \textit{i.e.} $\text{Ker } C = 0$ (which implies that the image of $C$ is all of $\mathcal{X}$, \textit{i.e.} $\text{Im } C = \mathcal{X}$.)

**endo-** endo means within, or internal, as in “endoscopy”. $C$ is an endomorphism if its codomain is the same as its domain, \textit{i.e.} $\mathcal{Y} = \mathcal{X}$.

**epi-** epi means on or upon, as in “epicenter”. $C$ is an epimorphism if the image of $C$ is the entire codomain, \textit{i.e.} $\text{Im } C = \mathcal{Y}$. If $C$ is an epimorphism it is described as being epic or onto or surjective.

**iso-** iso means equal, as in “isobar”. $C$ is an isomorphism if it maps each element of $\mathcal{X}$ to a unique element of $\mathcal{Y}$ and its image is the entire codomain, \textit{i.e.} $\text{Im } C = \mathcal{Y}$ and $\text{Ker } C = 0$. An isomorphism is a map that is both injective and surjective.

**mono-** mono means single, as in “monograph”. $C$ is a monomorphism if it maps each element of $\mathcal{X}$ to a unique element of $\mathcal{Y}$, \textit{i.e.} $\text{Ker } C = 0$. If $C$ is a monomorphism, $C$ is described as being monic or into or injective.