

Some Notes on Morphisms

On page 8 of the text, the author writes: “Essential to any grasp of algebra is a command of Greek prefixes.” Let’s look at the prefixes used in the text to describe a map $C : \mathcal{X} \rightarrow \mathcal{Y}$. Recall that \mathcal{X} is called the domain of C and \mathcal{Y} is called the codomain of C . Recall also that the kernel of C is the set of all elements of the domain that C maps to zero, and the image of C is the set of all elements of the codomain that can be expressed as Cx for some $x \in \mathcal{X}$.

auto- auto means self, as in “autobiography”. C is an automorphism if (i) its codomain is the same as its domain, *i.e.* $\mathcal{Y} = \mathcal{X}$, and (ii) the kernel of the map C is trivial, *i.e.* $\text{Ker } C = 0$ (which implies that the image of C is all of \mathcal{X} , *i.e.* $\text{Im } C = \mathcal{X}$.)

endo- endo means within, or internal, as in “endoscopy”. C is an endomorphism if its codomain is the same as its domain, *i.e.* $\mathcal{Y} = \mathcal{X}$.

epi- epi means on or upon, as in “epicenter”. C is an epimorphism if the image of C is the entire codomain, *i.e.* $\text{Im } C = \mathcal{Y}$. If C is an epimorphism it is described as being epic or onto or surjective.

iso- iso means equal, as in “isobar”. C is an isomorphism if it maps each element of \mathcal{X} to a unique element of \mathcal{Y} and its image is the entire codomain, *i.e.* $\text{Im } C = \mathcal{Y}$ and $\text{Ker } C = 0$. An isomorphism is a map that is both injective and surjective.

mono- mono means single, as in “monograph”. C is a monomorphism if it maps each element of \mathcal{X} to a unique element of \mathcal{Y} , *i.e.* $\text{Ker } C = 0$. If C is a monomorphism, C is described as being monic or into or injective.