

Technical Report WINLAB-TR-229

**OPTIMAL SIR-BASED POWER CONTROL IN
3G WIRELESS CDMA NETWORKS**

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ABSTRACT

One aspect of the wireless communications power control problem that has received little attention is the design of control algorithms that explicitly consider the tradeoff between the costs of transmit power and SIR error. In this paper, we propose a suboptimal control algorithm that explicitly considers these tradeoffs. We define costs for each mobile consisting of weighted sums of power, power increment, and SIR error, then derive corresponding optimal control laws, assuming changes in interference from one measurement to the next are small. We propose controllers based on these optimal control laws. The resulting control laws are technically suboptimal; however, simulation results for a three-user gain matrix show that when tradeoffs between SIR error and power usage are permitted, significant reductions in power usage are possible compared to the power balancing algorithm (PBA).

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1. Introduction

Because each user of a wireless communications system contributes to the interference affecting other users, effective and efficient power control strategies are essential for achieving both quality of service (QoS) and system capacity objectives.

Closed loop power control is used in wireless communication networks to compensate for time-varying channel characteristics, and to reduce mobile battery power consumption. The closed loop control structure for IS-95 (one of the currently implemented CDMA standards used in wireless networks) consists of an outer loop algorithm that updates the SIR threshold every 10 ms and an inner loop which calculates required powers based on SIR measurements updated at 800 Hz [20]. A block diagram illustrating the power control structure [21] is shown in Figure 1.1.

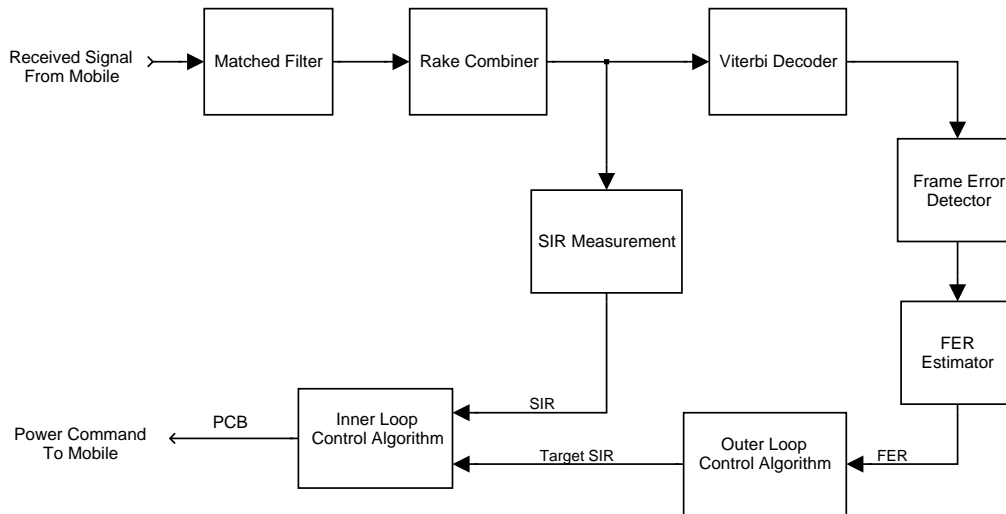


Figure 1.1: Block Diagram for Implementation of Power Control in CDMA Systems

1.1 Review of the literature

A variety of control algorithms for control of power in wireless communication networks have been discussed in the literature. Many of these have been designed to optimize or shown to

approximately optimize various criteria. One of the most common approaches to closed-loop power control in wireless communication networks is SIR balancing, also called power balancing. The SIR balancing solution was originally derived for satellite communications by Aein [1] and Meyerhoff [12], and adapted for wireless communications by Nettleton [13] and Zander [24] and [25]. SIR balancing attempts to minimize the combined power usage of all cochannel users by choosing a power vector that causes every mobile in the system to meet but not exceed the SIR threshold requirement. Other researchers have designed call admission and/or call dropping algorithms to address the cases when this is not possible.

Another class of algorithms seek to solve a static optimization problem. The well known distributed constrained power control (DCPC) algorithm maximizes the minimum attained user SIR subject to maximum power constraints [6]. Other algorithms have been designed to minimize total power consumption in the presence of large-scale fading [26] or over a set of discrete available power levels [22]. Still others seek to minimize outage probability [15].

Dynamic optimization has been used to minimize total combined mobile power consumption by formulating power control for log-normal fading channels in a stochastic framework [7], [8]. It also has been used to adaptively optimize quantization of SIR error measurements [19].

An alternative framework for developing power control algorithms is based on game theory [10] or economic formulations requiring the specification of a utility or cost function [2], [9], [11]. Various utility functions have been suggested [9], [18], [23]. The use of pricing to promote efficiency and fairness has been discussed extensively [16], [17].

Despite the variety of methods found in the literature, there has been little attention to dynamic optimal power control strategies that exploit the tradeoff between SIR and power usage. In particular, there has been no attempt to design such an optimal controller for the individual mobile transmitter so as to address power control in a distributed manner.

1.2 Interference Model

We consider here uplink (reverse link) power control. Although signature sequences are chosen to be orthogonal to one another, multipath distortion and asynchronous transmissions result in the presence of interference from other users to the signal received from a particular mobile at the base station. We designate by p_i the power transmitted by user i . We call the SIR of user i at the base station γ_i , and the background (receiver) noise power at the base station η . In considering the signal transmitted by the i th user, we denote the effective attenuation of the interfering signal from the j th user to the base station by g_{ij} , where $0 \leq g_{ij} \leq 1$, $\forall i$. In this

problem formulation, the interference at the base station to the i th user's signal is given by

$$I_i := \sum_{j \neq i} g_{ij} p_j + \eta \quad (1.1)$$

and the SIR of the i th user's signal is

$$\gamma_i := \frac{g_{ii} p_i}{I_i} = \frac{g_{ii} p_i}{\sum_{j \neq i} g_{ij} p_j + \eta}. \quad (1.2)$$

In the deterministic formulation of the power control problem for wireless networks, the noise power η is treated as constant.

2. Discrete Time Dynamic Optimal Control

We give here a very brief introduction to the aspects of discrete time dynamic optimization that we will use in the derivation. For a more detailed discussion see [3], [4], or [14].

Given a discrete time dynamic system with state x , input u , and state update function f , whose behavior is described by

$$x[k+1] = f(x[k], u[k], k) \quad (2.1)$$

and a cost function

$$J[N] := g(x[N]) + \sum_{k=0}^{N-1} L(x[k], u[k], k) \quad (2.2)$$

we define the discrete Hamiltonian

$$H[k] := L(x[k], u[k], k) + \lambda^T[k+1]f(x[k], u[k], k). \quad (2.3)$$

A control sequence $u[k]$ that produces a stationary value of the cost function must satisfy the discrete state-costate equations

$$x[k+1] = f(x[k], u[k], k) \quad (2.4)$$

$$\lambda[k] = H_x^T[k] \equiv L_x^T[k] + f_x^T[k]\lambda[k+1] \quad (2.5)$$

with

$$H_u[k] \equiv L_u[k] + \lambda^T[k+1]f_u[k] = 0, \quad (2.6)$$

where

$$L_x[k] := L_x(x[k], u[k], k).$$

This formulates the optimal control problem as a two-point boundary value problem with boundary conditions

$$x[0] = x_0 \quad (2.7)$$

$$\lambda[N] = g_x^T(x[N]). \quad (2.8)$$

The structure of the cost function $L(x[k], u[k], k)$ depends on the application. In our case, we are interested in deriving a control law for a discrete regulator based on a convex cost criterion.

We can consider the finite or infinite horizon case. In the finite horizon case, an appropriate quadratic final cost is

$$g(x[N]) = \frac{1}{2}e_f^T Q_f e_f \quad (2.9)$$

where the final error e_f is defined by

$$e_f = M_f x[N] - e^{tar} \quad (2.10)$$

with M_f being a weighting matrix and e^{tar} the target value. (Our target SIR error is zero.) Cost functions $L(x[k], u[k], k)$ that are quadratic in the state and the input are often chosen for the tractability of the solutions to the state-costate equations (2.4) and (2.5).

When satisfied along the entire path, the following pair of conditions together constitute a sufficient condition for local optimality:

$$\begin{bmatrix} H_{xx} & H_{xu} \\ H_{xu}^T & H_{uu} \end{bmatrix} > 0 \quad (2.11)$$

$$H_{uu} > 0. \quad (2.12)$$

Solutions to optimal control problems may be obtained using either a forward or a backward computational method. Implementing backward methods in real-time is generally not feasible. We use a forward solution method below to obtain controllers for the distributed mobile power control problem. As only the transient and not the steady state solutions depend on the initial conditions, the initial conditions can be arbitrarily chosen. For convenience, we choose zero initial powers. A more efficient approach would be to choose the initial powers to satisfy the static Nash optimality condition [10]. In fact, it has been shown [5] that the so long as the system is controllable and observable and the time-varying coefficients of the system have limits as time tends to infinity, the optimal control exists and is time-invariant, and further that the solution iteratively obtained is asymptotic to this optimal solution. Since our system obviously satisfies these criteria, for any fixed set of transmitting mobiles in a cell, we can safely iteratively compute the solution.

3. Controller Derivation

We make a number of simplifications in order to propose a tractable controller for wireless power control. We assume that the interference does not change significantly from one measurement to the next. (This assumption is commonly made in the literature.) The assumption holds if the contribution of noise is large with respect to the other transmissions; it holds also when enough number of mobiles are present and operating in steady state so that the fraction of the interference due to the addition or subtraction of one mobile does not have a large effect. We view the system as distributed and design controllers for each mobile individually. Our mobile power controller does not consider the dynamics that may result from the response of other mobiles to changes in its own power.

The resulting controllers will, of course, be technically suboptimal, but we will show in simulation that given a cost function, they can provide a significant improvement over the power balancing algorithm.

3.1 Problem Formulation

We choose the SIR error as our system state and the power increment as the system input. Accordingly, defining the following quantities¹,

$$\gamma[k] := \frac{gp[k]}{I[k]} \quad (3.1)$$

$$e[k] := \gamma[k] - \gamma^{tar} \quad (3.2)$$

$$u[k] := p[k+1] - p[k] \quad (3.3)$$

we have the system

$$\begin{aligned} e[k+1] &= \frac{g(p[k] + u[k])}{I[k+1]} - \gamma^{tar} \\ &= e[k] + \frac{gu[k]}{I[k]} + \frac{gp[k+1](I[k] - I[k+1])}{I[k]I[k+1]} \end{aligned} \quad (3.4)$$

$$\approx e[k] + \frac{gu[k]}{I[k]}. \quad (3.5)$$

¹For simplicity of notation, we omit the subscript i from all quantities in the remainder of this paper.

Note that the approximation in (3.5) is justified so long as the change in interference from one sample to the next is much smaller than the product of the two consecutive interference values. This assumption that the interference changes only slightly from one sampling instant to the next is commonly made in the communications literature since it is known that capacity and performance of wireless networks are limited by the presence of large interference values.

A consequence of (3.1) and (3.2) is that

$$p[k] = \frac{I[k](e[k] + \gamma^{tar})}{g}. \quad (3.6)$$

We will present suboptimal controllers for three separate cost functions:

$$J_I(e[k], u[k], K) = \frac{1}{2} \sum_{k=0}^K (qe^2[k] + su^2[k]) \quad (3.7)$$

$$J_{II}(e[k], p[k], u[k], K) = \frac{1}{2} \sum_{k=0}^K (qe^2[k] + 2rp[k] + su^2[k]) \quad (3.8)$$

$$J_{III}(e[k], p[k], u[k], K) = \frac{1}{2} \sum_{k=0}^K (qe^2[k] + rp^2[k] + su^2[k]) \quad (3.9)$$

where in all cases we require $q, r, s > 0$. The first cost is very natural for the dynamic system described by (3.5); however, mobile battery life depends on the total power used — not just the power update. To address this deficiency we have formulated the second and third cost functions. Since power is a positive quantity, it is not necessary to square it in computing the cost and may only lead to excessively penalizing large powers. (Most users would settle for reduced battery life if that were the only way to get adequate SIR.) Note that by squaring the SIR error, we are indirectly penalizing excessive power consumption in the sense that if the SIR is better than we need for satisfactory QoS, we assign the same cost as if the SIR is too low by the same amount.

Because of the relationship (3.6) between power and SIR error, we can express the second and third costs in terms of SIR error, interference, and power update as follows:

$$J_{II}(e[k], I[k], u[k], K) = \frac{1}{2} \sum_{k=0}^K \left(qe^2[k] + \left(\frac{2rI[k]}{g} \right) e[k] + \left(\frac{2rI[k]}{g} \right) \gamma^{tar} + su^2[k] \right) \quad (3.10)$$

$$J_{III}(e[k], I[k], u[k], K) = \frac{1}{2} \sum_{k=0}^K \left(\tilde{q}[k]e^2[k] + 2\phi[k]\gamma^{tar}e[k] + \phi[k](\gamma^{tar})^2 + su^2[k] \right) \quad (3.11)$$

where we have defined the quantities

$$\phi[k] := \frac{rI^2[k]}{g^2} \quad (3.12)$$

and

$$\tilde{q}[k] := (q + \phi[k]). \quad (3.13)$$

From this we see that the effect of including the power terms in the cost is to rescale the weight on the quadratic term and add a linear term in SIR error. Accordingly we derive the optimal control laws corresponding to the second and third costs whereas that for the first can be found in many standard optimal control texts. For the purpose of determining necessary and sufficient conditions for local optimality, the linear terms can be subsumed into the quadratic terms by augmenting the state vector by one state which is constantly one. (This extra state is uncontrollable but asymptotically stable.) We will discuss this in more detail below.

In the following section, we derive, for fixed constant interference, the equations for the controller corresponding to the cost J_{II} and simply present the results that arise from similar derivations for the other two controllers. The derivations for these other controllers are included in Appendices A and B.

3.2 Optimal Controller — Cost Linear in Power

With constant interference $I[k] = I, \forall k$, the discrete Hamiltonian corresponding to J_{II} in (3.8) is

$$\begin{aligned} H(e[k], u[k], K) &= \frac{1}{2} \sum_{k=0}^K \left(qe^2[k] + \frac{rI}{g}(e[k] + \gamma^{tar}) + su^2[k] \right) + \lambda[k+1]e[k+1] \quad (3.14) \\ &= \frac{1}{2} \sum_{k=0}^K \left(qe^2[k] + \frac{rI}{g}(e[k] + \gamma^{tar}) + su^2[k] \right) + \lambda[k+1] \left(e[k] + \frac{gu[k]}{I} \right). \end{aligned}$$

The condition that characterizes a stationary point

$$\frac{\partial H(e[k], u[k], K)}{\partial u[k]} = 0 \quad (3.15)$$

specifies the optimal power

$$u^*[k] = -\frac{g\lambda[k+1]}{sI}. \quad (3.16)$$

The multiplier sequence $\lambda[k]$ is chosen such that

$$\lambda[k] = \frac{\partial H(e[k], u[k], K)}{\partial e[k]} \quad (3.17)$$

which after solving for $\lambda[k+1]$ yields

$$\lambda[k+1] = \lambda[k] - qe[k] - \frac{rI}{g}. \quad (3.18)$$

To simplify the equations that follow, we define the quantity

$$\psi[k] := \frac{sI^2[k]}{g^2}. \quad (3.19)$$

Now writing the Lagrange multiplier $\lambda[k]$ as ²

$$\lambda[k] = \mu[k]e[k] + \nu[k] \quad (3.20)$$

and substituting for $\lambda[k]$ in (3.18) yields

$$\lambda[k+1] = (\mu[k] - q)e[k] + \nu[k] - \frac{rI}{g} \quad (3.21)$$

but evaluating (3.20) at time step $k+1$ and substituting for $e[k+1]$ from (3.5) with $u[k] = u^*[k]$ yields

$$\begin{aligned} \lambda[k+1] &= \mu[k+1]e[k+1] + \nu[k+1] \\ &= \mu[k+1](e[k] - \psi^{-1}\lambda[k+1]) + \nu[k+1]. \end{aligned} \quad (3.22)$$

Isolating $\lambda[k+1]$ we have

$$\lambda[k+1] = \frac{\psi(\mu[k+1]e[k] + \nu[k+1])}{\psi + \mu[k+1]}. \quad (3.23)$$

Equating the right-hand sides of (3.21) and (3.23) and recalling that the $e[k]$ and $\nu[k]$ terms must separately sum to zero yields

$$\boxed{\mu[k+1] = \frac{\psi(\mu[k] - q)}{\psi - (\mu[k] - q)}} \quad (3.24)$$

and an expression for $\nu[k+1]$ in terms of $\nu[k]$, $\mu[k+1]$, $\gamma[k]$, and ψ . Substituting for $\mu[k+1]$ then yields the following expression for $\nu[k+1]$

$$\boxed{\nu[k+1] = \frac{\psi(g\nu[k] - rI)}{g(\psi - (\mu[k] - q))}}. \quad (3.25)$$

Using (3.23), we can substitute for $\lambda[k+1]$ in (3.16) to obtain

$$\boxed{u^*[k] = -\frac{g}{sI} \left((\mu[k] - q)e[k] + \nu[k] - \frac{rI}{g} \right)}. \quad (3.26)$$

Next, we show that the second order sufficient conditions for local optimality are satisfied. In our implementation, we have required $q > 0$, so $H_{uu} = q > 0$. $H_{eu} = 0$ so the matrix in (2.11) is block diagonal. Using the augmented state vector approach described above, we find that

$$H_{ee} = \begin{bmatrix} q & rI/g \\ rI/g & 2rI\gamma^{tar}/g \end{bmatrix}. \quad (3.27)$$

²In the case of quadratic costs, it can be shown that the expression for $\lambda[k]$ should be linear in the state $e[k]$. See, for example, Bryson and Ho [4].

This leads to the requirement that $2\gamma^{tar} > rI/g$, which can be viewed as prescribing a limit on the allowable interference, $I < 2\gamma^{tar}g/r$, or on the power cost weight r , namely $r < 2\gamma^{tar}g/I$, in the presence of a given level of interference.

Finally, we note that to implement a controller based on (3.24), (3.25), and (3.26), we need the initial values for the controller states μ and ν . These are derived as follows. It can be shown that the boundary condition (2.8) implies the following expression for $\lambda[0]$,

$$\lambda[0] = 2q + \frac{q^2}{s} \left(\frac{g}{I[N]} \right)^2 e[0] + \frac{qr}{s} \left(\frac{g}{I[N]} \right) + \frac{rI[N]}{g}. \quad (3.28)$$

From the form of (3.28) we see that our initial controller states should be

$$\mu[0] = 2q + \frac{q^2}{s} \left(\frac{g}{I[N]} \right)^2, \quad (3.29)$$

$$\nu[0] = 2q + \frac{qr}{s} \left(\frac{g}{I[N]} \right) + \frac{rI[N]}{g}. \quad (3.30)$$

We initialize our power commands to zero; however, any other initialization would change only the transient and not the steady-state values. Since the initial values of the controller states depend on the final values of the interferences, we need good initial estimates thereof.

The critical reader may have noticed that we mentioned using an augmented state vector $x[k] = [e[k], 1]$ in order to write the cost in terms of quadratic terms only. If we had used this augmented state vector in the derivation above, our Lagrange multiplier λ would have had two components; however, performing this more elaborate derivation, one finds that the second component of λ is superfluous, hence we have omitted it in order to make the presentation more readable.

Steady State Values

From (3.24) we see that the steady state value $\bar{\mu}$ of $\mu[k]$ must satisfy a quadratic equation

$$\bar{\mu}^2 - q\bar{\mu} - q\bar{\psi} = 0 \quad (3.31)$$

in which \bar{I} and $\bar{\psi}$ represent the steady state values of the interference and the parameter ψ respectively. Since $\bar{\mu}$ must be nonnegative and both q and $\bar{\psi}$ are positive we choose the positive square root of the solution

$$\bar{\mu} = \frac{1}{2}q \pm \frac{1}{2}\sqrt{q^2 + 4q\bar{\psi}}. \quad (3.32)$$

From (3.25) we determine that

$$\bar{\nu} = \frac{r\bar{I}\bar{\psi}}{g(\bar{\mu} - q)}. \quad (3.33)$$

3.3 Optimal Controller — Cost Quadratic in Power

A similar derivation yields for the third cost J_{III} ,

$$\mu[k+1] = \frac{\psi(\mu[k] - q - \phi)}{\psi - (\mu[k] - q - \phi)} \quad (3.34)$$

and

$$\nu[k+1] = \frac{\psi(\nu[k] - \phi\gamma^{tar})}{\psi - (\mu[k] - q - \phi)} \quad (3.35)$$

with

$$u^*[k] = -\frac{g}{sI} ((\mu[k] - q - \phi)e[k] + \nu[k] - \phi\gamma^{tar}). \quad (3.36)$$

The initial value of the Lagrange multiplier is

$$\lambda[0] = \left(2q + \frac{q^2 g^2}{sI[N]}\right) e[0] + \frac{qr\gamma^{tar}}{s} + \phi[N]\gamma^{tar} \quad (3.37)$$

and the steady state values are obtained from

$$\bar{\mu} = \frac{1}{2}(q + \bar{\phi}) \pm \frac{1}{2}\sqrt{(q + \bar{\phi})^2 + 4\bar{\psi}(q + \bar{\phi})} \quad (3.38)$$

and

$$\bar{\nu} = \frac{\gamma^{tar}\bar{\phi}\bar{\psi}}{\bar{\mu} - q - \bar{\phi}}. \quad (3.39)$$

The requirement that H_{ee} be positive definite reduces to the requirement that $q + \phi > \phi, \forall k$, so no restriction on acceptable interference or weight values arises.

3.4 Optimal Controller — Cheap Power Case

When we use the cost J_I , neglecting the cost of power, the controller obtained is characterized by

$$\mu[k+1] = \frac{\psi(\mu[k] - q)}{\psi - (\mu[k] - q)} \quad (3.40)$$

and

$$\nu[k+1] = \frac{\psi\nu[k]}{\psi - (\mu[k] - q)} \quad (3.41)$$

whence

$$u^*[k] = -\frac{g((\mu[k] - q)e[k] + \nu[k])}{sI} \quad (3.42)$$

with ψ again defined by (3.19).

The initial value of the Lagrange multiplier is

$$\lambda[0] = \frac{q(2\psi[N] + q)e[0]}{\psi[N] + q} \quad (3.43)$$

and the steady state value of μ is obtained from

$$\bar{\mu} = \frac{1}{2}q \pm \frac{1}{2}\sqrt{q^2 + 4q\bar{\psi}}. \quad (3.44)$$

Since (3.44) shows that $\bar{\mu} \neq 0$, inspection of (3.41) indicates that the steady state value $\bar{\nu}$ must be zero.

4. Controller Implementation

The derivations above are analogous to the transition matrix solution method [3] which has the advantage that iteration is not required. However, the performance of any of these controllers will depend on the choice of initial values for the controller states. Again, strictly optimal performance would require these values to be known exactly.

In practical application, mobiles would enter a cell in which a preexisting level of interference was present. So long as the change in level of interference observed by the mobile in response to its broadcasts were not large, a reasonable method for initializing the controller states would be to calculate the value of $\lambda[0]$ corresponding to the interference observed upon entering the cell.

The three controllers have been implemented in MATLAB simulations. To verify the coding, the simulations were run to obtain steady state values for the controller states μ and ν . These values were found to match those calculated analytically using (3.32), (3.33), (3.38), (3.39), and (3.44).

5. Simulation

We simulated controller performance for all three of the controller designs using both MATLAB scripts and SIMULINK block diagrams. The SIMULINK block diagrams illustrate the structure of the controller and its relation to the rest of the system and allow the user to generate C-language source code for the control algorithm. The MATLAB scripts, on the other hand, run faster, so most of the simulation results below were obtained using MATLAB scripts.

The controllers were tested under a variety of conditions. Gain matrices were varied, as were number of mobiles and cost weights. In all cases, the SIR converged much faster than the power.

5.1 Simulink Model

A model of power control for a three mobile system, constructed in SIMULINK, is shown in figure 5.1. The block diagram for the individual mobile is shown in Figure 5.2. The Inner Loop Control Algorithm block of Figure 1.1 is implemented in the Controller block of the SIMULINK model, which is shown in Figure 5.3.

5.2 Examples of Controller Performance

The plots below present controller performance in simulation. For illustration purposes, a three mobile system was considered. The attenuation matrix G was chosen such that the off-diagonal entries (corresponding to interference from other mobiles) were two to three orders of magnitude smaller than the diagonal entries (mobile's own attenuation) which were chosen to be close to 1. Noise power of 0.01 was used.

In Figures 5.4 through 5.6, examples of controller behavior are shown for the controller corresponding to linear power cost J_{II} without initialization of the controller states. It can be seen that even without proper initialization, we have reasonably fast convergence. In addition, these figures illustrate the effects of changing the cost weights.

As expected, in Figure 5.4 we see that decreasing the SIR error cost weight leads to lower powers and larger SIR errors. In Figure 5.5, increasing the power cost weight r leads to lower

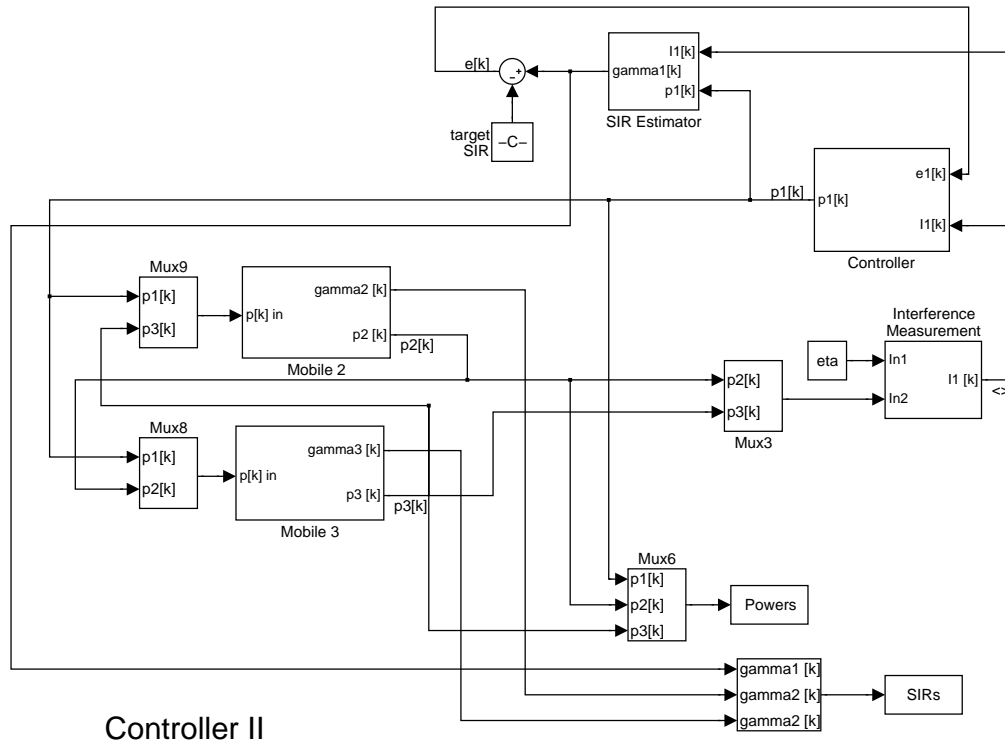


Figure 5.1: Block diagram of the three mobile suboptimal power control simulation

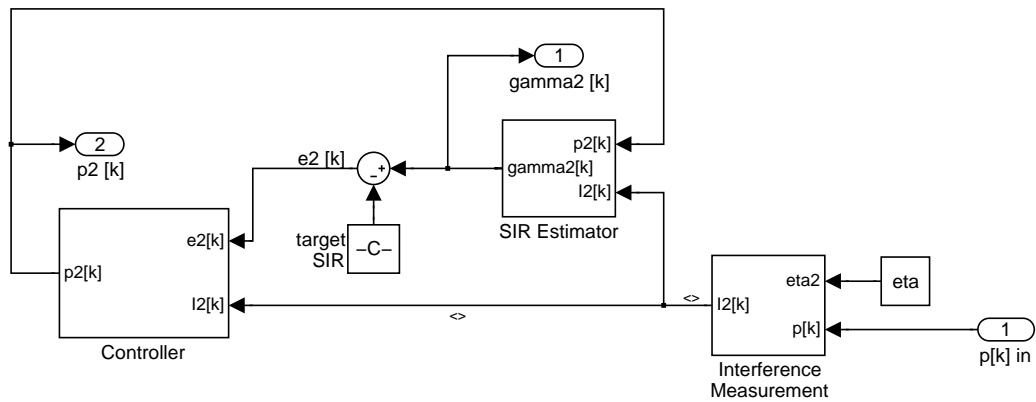


Figure 5.2: Block diagram of individual mobile suboptimal power control model

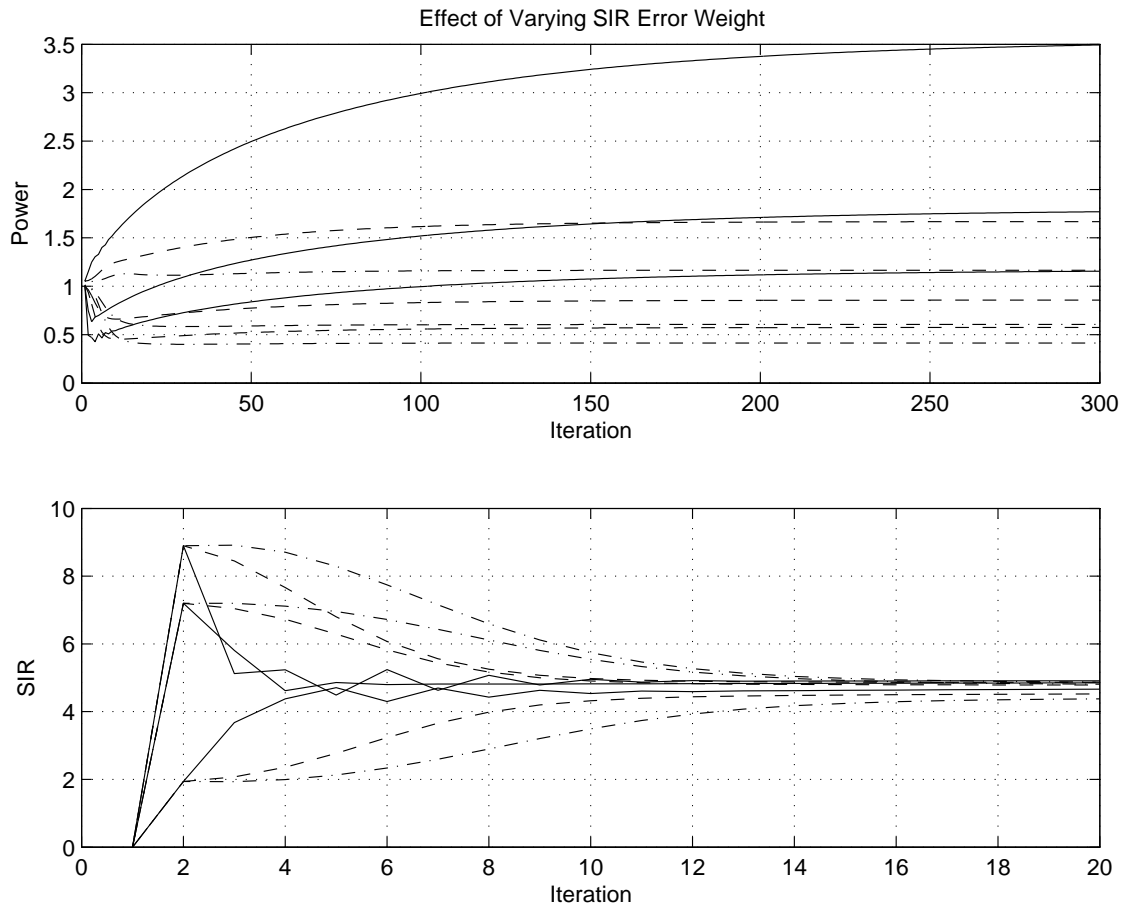


Figure 5.4: Suboptimal Controller Corresponding to J_{II} , $\eta = 0.01$

| | | |
|-----------------|-----------|-------------|
| | line type | (q,r,s) |
| Figure 5.4 Key: | ----- | (5,1,100) |
| | ----- | (1,1,100) |
| | - . . . - | (0.5,1,100) |

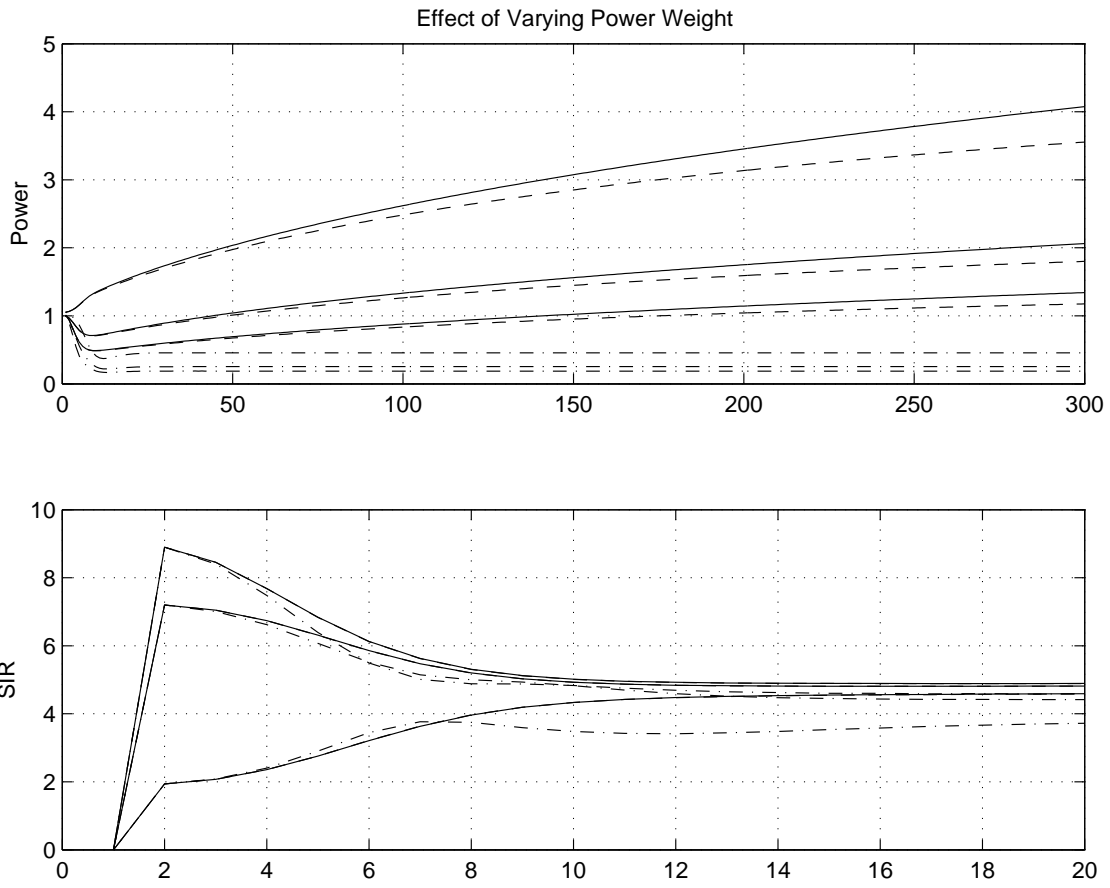


Figure 5.5: Suboptimal Controller Corresponding to J_{II} , $\eta = 0.01$

Figure 5.5 Key:

| | |
|-----------|---------------|
| line type | (q,r,s) |
| ----- | (1,0.001,500) |
| - - - - | (1,0.1,500) |
| - . . . - | (1,10,500) |

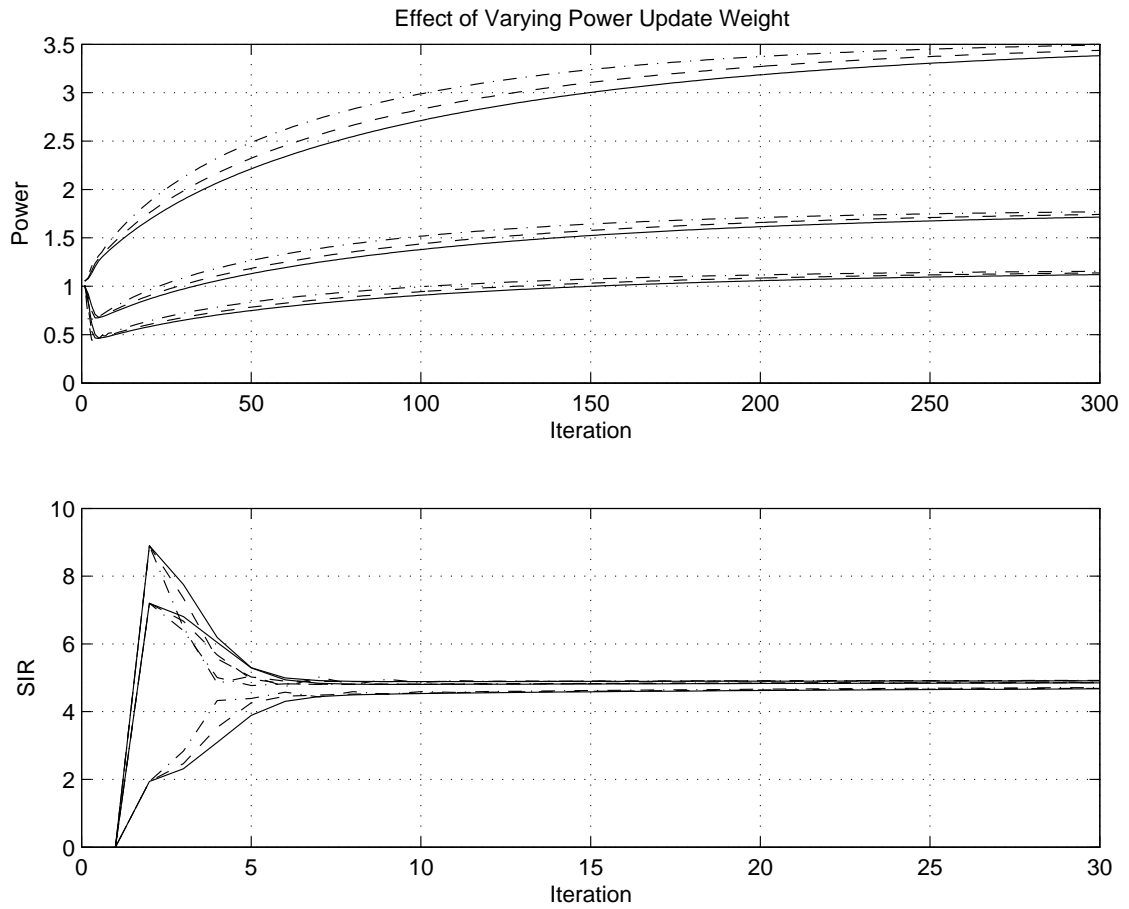


Figure 5.6: Suboptimal Controller Corresponding to J_{II} , $\eta = 0.01$

Figure 5.6 Key:

| | |
|-----------|-------------|
| line type | (q,r,s) |
| ----- | (1,0,2,200) |
| - . - . - | (1,0,2,150) |
| ----- | (1,0,2,100) |

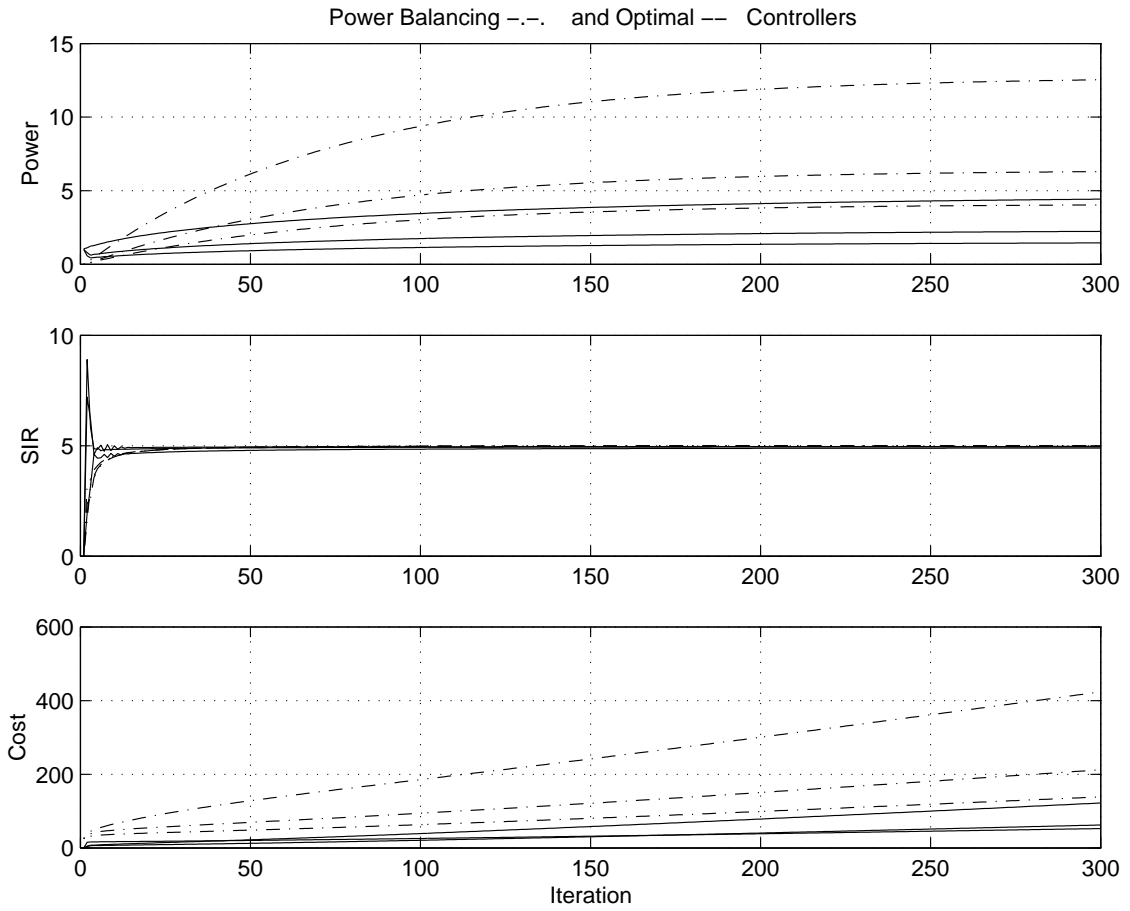


Figure 5.7: Comparison between power balancing (dashed lines) and suboptimal controller with linear power weight (solid lines), $\eta = 0.01$

6. Conclusion

We have designed several suboptimal controllers using different cost functions. We have then tested the controller designed for cost linear in power and quadratic in SIR error and power update in simulation. We have thereby demonstrated the potential of the resulting controller strategy to save power and improve QoS as compared with power balancing when tradeoffs between SIR error and power usage are permitted. Additional research topics of potential interest include choice of cost function and choice of interference update rate.

A. Optimal Control with Weighted SIR Error and Power Increment

We define the following quantities:

$$\gamma[k] := \frac{gp[k]}{I[k]} \quad (\text{A.1})$$

$$e[k] := \gamma[k] - \gamma^{tar} \quad (\text{A.2})$$

$$u[k] := p[k+1] - p[k]. \quad (\text{A.3})$$

We will obtain the optimal $u[k]$ for the feedback controller for the system with state $e[k]$ and input $u[k]$.

Since

$$p[k+1] = p[k] + u[k] \quad (\text{A.4})$$

we obtain

$$e[k+1] = \frac{g(p[k] + u[k])}{I[k+1]} - \gamma^{tar} \quad (\text{A.5})$$

$$= e[k] + \frac{gu[k]}{I[k]} + \frac{gp[k+1](I[k] - I[k+1])}{I[k]I[k+1]} \quad (\text{A.6})$$

$$\approx e[k] + \frac{gu[k]}{I[k]}. \quad (\text{A.7})$$

We define the individual cost function

$$J(e, u, K) = \frac{1}{2} \sum_{k=0}^K (qe^2[k] + tu^2[k]) + h(K), \quad q, t > 0 \quad (\text{A.8})$$

and for simplicity take the end cost $h(K) = 0$. The Hamiltonian is then

$$H(e[k], u[k], K) = \frac{1}{2} \sum_{k=0}^K (qe^2[k] + tu^2[k]) + \lambda[k+1]e[k+1] \quad (\text{A.9})$$

$$= \frac{1}{2} \sum_{k=0}^K (qe^2[k] + tu^2[k]) + \lambda[k+1] \left(e[k] + \frac{gu[k]}{I[k]} \right). \quad (\text{A.10})$$

The condition that

$$\frac{\partial H(e[k], u[k], K)}{\partial u[k]} = 0 \quad (\text{A.11})$$

leads to

$$tu^*[k] + g \frac{\lambda[k+1]}{I[k]} = 0$$

so the optimal power is

$$u^*[k] = -\frac{g\lambda[k+1]}{tI[k]}. \quad (\text{A.12})$$

Next we require

$$\lambda[k] = \frac{\partial H(e[k], u[k], K)}{\partial e[k]} \quad (\text{A.13})$$

which implies that

$$\lambda[k] = qe[k] + \lambda[k+1]. \quad (\text{A.14})$$

Substituting for the optimal power $u^*[k]$ from (A.12) into (A.5) we have

$$e[k+1] = e[k] - \frac{g^2\lambda[k+1]}{tI^2[k]}. \quad (\text{A.15})$$

Solving (A.14) for $\lambda[k+1]$ yields

$$\lambda[k+1] = \lambda[k] - qe[k]. \quad (\text{A.16})$$

Now writing

$$\lambda[k] = P[k]e[k] + s[k] \quad (\text{A.17})$$

and substituting for $\lambda[k]$ in (A.16) yields

$$\lambda[k+1] = (P[k] - q)e[k] + s[k] \quad (\text{A.18})$$

but evaluating (A.17) at time step $k+1$ and substituting for $e[k+1]$ from (A.15) yields

$$\lambda[k+1] = P[k+1]e[k+1] + s[k+1] \quad (\text{A.19})$$

$$= P[k+1] \left(e[k] - \frac{g^2\lambda[k+1]}{tI^2[k]} \right) + s[k+1]. \quad (\text{A.20})$$

Isolating $\lambda[k+1]$ we have

$$\lambda[k+1] = \frac{\psi[k] (s[k+1] + P[k+1]e[k+1])}{P[k+1] + tI^2[k]} \quad (\text{A.21})$$

where we have defined

$$\psi[k] := \frac{tI^2[k]}{g^2}. \quad (\text{A.22})$$

So equating the right-hand sides of (A.18) and (A.21) and noting that the $e[k]$ and $s[k]$ terms must separately sum to zero yields

$$e[k] \left((P[k] - q) - \frac{P[k+1]tI^2[k]}{g^2P[k+1] + tI^2[k]} \right) = 0 \quad (\text{A.23})$$

and

$$s[k] = \frac{tI^2[k]}{g^2P[k+1] + tI^2[k+1]} s[k+1]. \quad (\text{A.24})$$

Now solving (A.23) for $P[k + 1]$ yields

$$\boxed{P[k + 1] = \frac{\psi[k] (P[k] - q)}{\psi[k] - (P[k] - q)}} \quad . \quad (\text{A.25})$$

Substituting for $P[k + 1]$ in (A.24) yields the following expression for $s[k + 1]$

$$\boxed{s[k + 1] = \frac{\psi[k] s[k]}{\psi[k] - (P[k] - q)}} \quad . \quad (\text{A.26})$$

Using (A.21), we can substitute for $\lambda[k + 1]$ in (A.12) to obtain

$$\boxed{u^*[k] = -\frac{g((P[k] - q)e[k] + s[k])}{tI[k]}} \quad . \quad (\text{A.27})$$

Steady State Values

The steady state value \bar{P} of $P[k]$ must satisfy the quadratic equation

$$\bar{P}^2 g^2 - \bar{P} q g^2 - q t \bar{I}^2 = 0 \quad (\text{A.28})$$

where \bar{I} represents the steady state value of the interference. The expression for \bar{P} is thus

$$\bar{P} = \frac{1}{2} q \pm \frac{1}{2} \sqrt{q^2 + 4q\bar{\psi}} \quad (\text{A.29})$$

where $\bar{\psi}$ represents the steady state value of the dimensionless quantity ψ . Since (A.29) shows that $\bar{P} \neq 0$, inspection of (A.26) indicates that, the steady state value \bar{s} must be zero.

B. Optimal Control with Cost Quadratic in Power

We define the following quantities:

$$\gamma[k] := \frac{gp[k]}{I[k]} \quad (\text{B.1})$$

$$e[k] := \gamma[k] - \gamma^{tar} \quad (\text{B.2})$$

$$u[k] := p[k+1] - p[k]. \quad (\text{B.3})$$

We will obtain the optimal $u[k]$ for the feedback controller for the system with state $e[k]$ and input $u[k]$.

Since

$$p[k+1] = p[k] + u[k] \quad (\text{B.4})$$

we obtain

$$e[k+1] = \frac{g(p[k] + u[k])}{I[k+1]} - \gamma^{tar} \quad (\text{B.5})$$

$$= e[k] + \frac{gu[k]}{I[k]} + \frac{gp[k+1](I[k] - I[k+1])}{I[k]I[k+1]} \quad (\text{B.6})$$

$$\approx e[k] + \frac{gu[k]}{I[k]}. \quad (\text{B.7})$$

Also, from (B.1) and (B.2) we can write

$$p[k] = \frac{I[k](e[k] + \gamma^{tar})}{g}. \quad (\text{B.8})$$

We define the individual cost function

$$J(e, u, K) = \frac{1}{2} \sum_{k=0}^K (qe^2[k] + rp^2[k] + tu^2[k]) + h(K), \quad q, r, t > 0 \quad (\text{B.9})$$

and for simplicity take the end cost $h(K) = 0$. The Hamiltonian is then

$$H(e[k], u[k], K) = \frac{1}{2} \sum_{k=0}^K \left(qe^2[k] + \frac{rI^2[k](e[k] + \gamma^{tar})^2}{g^2} + tu^2[k] \right) + \lambda[k+1]e[k+1] \quad (\text{B.10})$$

$$= \frac{1}{2} \sum_{k=0}^K \left(qe^2[k] + \frac{rI^2[k](e[k] + \gamma^{tar})^2}{g^2} + tu^2[k] \right) + \lambda[k+1] \left(e[k] + \frac{gu[k]}{I[k]} \right). \quad (\text{B.11})$$

The condition that

$$\frac{\partial H(e[k], u[k], K)}{\partial u[k]} = 0 \quad (\text{B.12})$$

specifies the optimal power

$$u^*[k] = -\frac{g\lambda[k+1]}{tI[k]}. \quad (\text{B.13})$$

Next we require

$$\lambda[k] = \frac{\partial H(e[k], u[k], K)}{\partial e[k]}. \quad (\text{B.14})$$

We define the dimensionless quantities

$$\phi[k] := \frac{rI^2[k]}{g^2} \quad (\text{B.15})$$

$$\psi[k] := \frac{tI^2[k]}{g^2}. \quad (\text{B.16})$$

After solving for $\lambda[k+1]$, (B.14) yields

$$\lambda[k+1] = \lambda[k] - qe[k] - \phi[k] (e[k] + \gamma^{tar}). \quad (\text{B.17})$$

Now writing

$$\lambda[k] = P[k]e[k] + s[k] \quad (\text{B.18})$$

and substituting for $\lambda[k]$ in (B.17) yields

$$\lambda[k+1] = (P[k] - q)e[k] + s[k] - \phi[k] (e[k] + \gamma^{tar}) \quad (\text{B.19})$$

but evaluating (B.18) at time step $k+1$, substituting for $e[k+1]$ from (B.7), letting $u[k] = u^*[k]$, and isolating $\lambda[k+1]$ we have

$$\lambda[k+1] = \frac{\psi[k](P[k+1]e[k] + s[k+1])}{\psi[k] + P[k+1]}. \quad (\text{B.20})$$

Equating the right-hand sides of (B.19) and (B.20) and noting that the $e[k]$ and $s[k]$ terms must separately sum to zero yields

$$P[k+1] = \frac{\psi[k](P[k] - q - \phi[k])}{\psi[k] - (P[k] - q - \phi[k])} \quad (\text{B.21})$$

and an expression for $s[k+1]$ in terms of $s[k]$, $P[k]$, $\gamma[k]$, $\phi[k+1]$, and $\psi[k]$. Substituting for $P[k+1]$ then yields the following expression for $s[k+1]$

$$s[k+1] = \frac{\psi[k](s[k] - \phi[k]\gamma^{tar})}{\psi[k] - (P[k] - q - \phi[k])}. \quad (\text{B.22})$$

Using (B.20), we can substitute for $\lambda[k+1]$ in (B.13) to obtain

$$u^*[k] = -\frac{g}{tI[k]} ((P[k] - q - \phi[k])e[k] + s[k] - \phi[k]\gamma^{tar}). \quad (\text{B.23})$$

Steady State Values

The steady state value \bar{P} of $P[k]$ must satisfy a quadratic equation in q , \bar{I} and $\bar{\psi}$, the last two representing the steady state values of the interference and the dimensionless parameter ψ respectively. Solving this quadratic we obtain an expression for \bar{P} ,

$$\bar{P} = \frac{1}{2}(q + \bar{\phi}) \pm \frac{1}{2}\sqrt{(q + \bar{\phi})^2 + 4\bar{\psi}(q + \bar{\phi})} \quad (\text{B.24})$$

Similarly, we find that

$$\bar{s} = \frac{\gamma^{tar}\bar{\phi}\bar{\psi}}{\bar{P} - q - \bar{\phi}}. \quad (\text{B.25})$$

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