Final Exam

1. (20 points)
   
   (a) State the criterion for determining whether an additive plant perturbation affects the stability of the closed-loop system.

   (b) State the criterion for determining whether an multiplicative plant perturbation affects the stability of the closed-loop system.
(c) Use the appropriate criterion to determine whether the unmodelled pole $2/(s + 2)$ of the plant affects the stability of the nominal plant

$$G(s) = \frac{30}{(s + 1)(s + 5)}.$$
2. (15 points) The Bode plot for the open loop transfer function

\[ G(s) = \frac{20}{s^2} \]

is shown below. The values of \( \alpha \), \( \omega_m \), and \( \tau \) are required for a lead compensator

\[ G_c(s) = \frac{(1 + \alpha \tau s)}{\alpha(1 + s)} \]

to increase the phase margin.
(a) Suppose $\zeta = 0.2$ is required. Determine the appropriate value for $\alpha$.

(b) Suppose $\alpha = 20$. From the Bode plot, determine the value of $\omega_m$. (Justify your answer.)

(c) For this value of $\omega_m$, what is the value of $\tau$?
3. (15 points) Consider the system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
3 & 0 \\
1 & 2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u
\]

with output

\[ y = [1 \ 0] \mathbf{x}. \]

Calculate the controllability matrix \( P_c \) and the observability matrix \( P_o \). State whether the system is controllable or not and whether it is observable or not.
4. (15 points) Consider the system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix} 3 & 0 \\
1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\
x_2 \end{bmatrix} + \begin{bmatrix} 0 \\
1 \end{bmatrix} u
\]

with output

\[ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + u. \]

Determine the poles of the observer if the observer gain is \( L = [0 \ 1]^T \).
5. (15 points) Consider the system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
3 & 0 \\
1 & 2
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} u
\]

with output

\[y = [1, 1] x + u.\]

Find the transfer function for the system.
6. (15 points) Consider the negative unity feedback system with open–loop transfer function

\[ G(s) = \frac{Ks + 1}{s(s + p_1)(s + p_2)}. \]

If \(1 < p_1 < 5\), \(-1 < p_2 < 2\), and \(K = 3\), identify the four worst–case polynomials for the closed–loop transfer function and determine whether the system is stable for all variations of the parameters within the indicated ranges.