Exam 3

1. (10 points) The forward path transfer function of a unity negative feedback system

\[ G(s) = \frac{\beta}{(s + 1)(s^2 + \beta s + 1)}. \]

To find the root locus for \( \beta \), we must obtain an equation of the form \( 1 + \beta G_1(s) = 0 \). Determine \( G_1(s) \).
2. (30 points) From the three plots given on the next page, select the correct root locus plot for the unity negative feedback system with open loop transfer function

\[ G(s) = \frac{Ks(s + 3)}{(s + 1)(s^2 + 4s + 5)}. \]

For each plot that you do not select, indicate how you determined that it is not correct. You may find it helpful (both to determine the correct plot and to obtain partial credit if your choice is incorrect) to answer the following questions.

(a) What are the poles and zeros of the transfer function?

(b) What are the angles of the asymptotes and the value of the intercept of the asymptotes with the real axis? (First give the expression for the quantities, then do the calculations.)

(c) What are the angles of departure of the loci from the complex poles? (First give the expression for the quantity, then do the calculation.)
(d) To the right of each graph, indicate whether it is the root locus plot of the transfer function

\[ G(s) = \frac{K s (s + 3)}{(s + 1)(s^2 + 4s + 5)} \]

or why it is not.
3. (25 points) Match the Bode plots to the transfer functions. (Write the letter of the transfer function in the margin to the left or right of the corresponding plot.)

\[ G_a(s) = \frac{1}{s^2 + 110s + 1000} \]
\[ G_b(s) = \frac{s + 10}{s^2 + 101s + 100} \]
\[ G_c(s) = \frac{s + 100}{s^2 + 11s + 10} \]
\[ G_d(s) = \frac{s^2 + 33s + 90}{s^2 + 110s + 1000} \]
\[ G_e(s) = \frac{s + 30}{s + 100} \]
4. (15 points) Match the Nyquist plots to the open loop transfer functions. (Write the letter of the transfer function in the margin to the left or right of the corresponding plot. Show appropriate calculations to explain your choice in the space to the right of the plot.)

\[ G_a(s) = \frac{s - 3}{s^3 + 3s^2 + 3s + 1} \], \quad G_b(s) = \frac{20}{s^3 + 3s^2 + 3s + 1} \], \quad G_c(s) = \frac{8}{s^3 + 3s^2 + 3s + 1} \]
5. (10 points) The root locus plot below corresponds to the open loop transfer function \( KG(s) \). Circle the correct Bode plot for this open loop system (justify your choice) and determine the value of the gain \( K \) from the graph.
6. (10 points) The root locus plot below corresponds to the open loop transfer function $KG(s)$. Circle the correct Nyquist plot for this system (justify your choice) and determine the value of the gain margin from the graph.