Nonlinear Constrained Component Optimization of a Plug-in Hybrid Electric Vehicle Powertrain

Emrah T. Yildiz¹, Quazi Farooqi², Sohel Anwar³, Yaobin Chen⁴, and Afshin Izadian⁵

¹Cummins, Inc., Columbus, IN, USA
²Graduate Student, Mechanical Engineering, IUPUI, Indianapolis, USA
³Associate Professor, Dept. of Mechanical Engineering, IUPUI, USA
⁴Professor and Chair, Dept. of Electrical and Computer Engineering, IUPUI, USA
⁵Assistant Professor, Dept. of Engineering Technology, IUPUI, USA
E-mail: soanwar@iupui.edu

Abstract

Transportation is rapidly evolving all around the world to meet the mandatory emission regulations and to find alternative solutions for high fossil fuel prices. Electrically propelled vehicles are one of the most promising solutions among all alternatives offered in terms of reliability, availability, feasibility, and safety. Transportation electrification can be accomplished in forms of a Hybrid Electric Vehicle (HEV) or an All Electric Vehicle (EV). However, the shortcomings of a fully electric vehicle in fulfilling all performance requirements make the plug-in hybrid electric vehicle (PHEV), the most feasible propulsion system. In PHEVs the optimal combination of the properly sized components such as internal combustion engine, electric motor, and energy storage unit are crucial to meet the performance requirements, improve fuel efficiency, and obtain cost effectiveness.

This paper demonstrates a novel application of Particle Swarm Optimization (PSO) approach to optimally size the vehicle powertrain components (e.g. engine power, electric motor power, battery energy capacity) while meeting the critical performance requirements. Compared to conventional optimization methods, PSO has been proven an efficient and precise optimization method for nonlinear constrained optimization problems. The PHEV powertrain configuration with the optimized components was used to form a new vehicle model in Powertrain System Analysis Toolkit (PSAT). Simulation results demonstrate that the optimized vehicle components using PSO results in higher operation performance and more fuel efficient vehicle.

Keywords: Optimal Component Sizing, Plug-in Hybrid Electric Vehicle, Particle Swarm Optimization, PSAT

1. Introduction

The idea of an electrically propelled vehicle was first introduced by Prof. Ferdinand Porsche in 1899. Several configurations were proposed based on the original concept by manufacturers for many years. However, because of low fuel prices and virtually no emission regulations this technology was not a center of interest after the early development period for a long time. However, in 1990s, researchers and manufacturers started intensely leaning on improving the electric vehicle technology to provide a highly fuel-efficient and significantly low emission transportation solution to ever-increasing cost of fuel and first generation of emission requirements. Plug-in hybrid electric vehicle (PHEV) is a modified version of an HEV in which the vehicle has a larger battery pack that can be charged by external sources e.g. home electric outlets, and by internal sources such as regenerative braking, and an engine driven generator. To minimize the consumed fuel by gasoline engine, and to propel the vehicle with electric energy storage systems (ESS), they should be utilized more frequently and charged by plug-in charging capability. Recent improvements in battery technology to hold higher amount of energy per unit of weight, has made the plug in hybrid electric vehicles a reality. Thus, PHEVs hold the promise to further improve the energy efficiency and reduce the environmental cost of vehicles.
While a lot of research has been done in the field of HEVs, there has been little work on PHEV powertrain, particularly with respect to optimal component sizing. Since the fuel consumption, emission levels and performance requirements depend on the powertrain components and configuration of these vehicles, component sizing was one of the areas that received attention in the recent past.

In [1], V. Galdi used a genetic algorithm based methodology to size major components of an HEV. The research aimed to minimize a function objective, which took into account the technical specifications, and environmental, social, and economic aspects. Xiaolan Wu, in [2], optimized the size of components in Parallel HEVs using particle swarm optimization technique. Another Parallel HEV powertrain component sizing optimization was done by Wenzhong Gao [3]. He used global optimization algorithms, DIRECT (Divided RECTangles), simulated annealing, and genetic algorithm and compared the results of those three.

In [4], Zhu Zhengli presented research results on powertrain design by optimal sizing of a series HEV using an adaptive based hybrid genetic algorithm. Similarly, Xudong Liu in [5], using a hybrid genetic algorithm, searched for the optimal sizes of components for a series HEV.

To obtain electric propulsion more frequently in a hybrid electric vehicle, the capacity of the energy storage device should increase, which consequently increases the cost and mass of the vehicle. Larger ESS may not generally be fully charged during regenerative braking, thus would require being plugged-in to an electrical outlet for additional charging, particularly when the available configuration is required to provide a 40-mile driving range and to maintain the vehicle performance requirements. Thus, a Plug-in Hybrid Electric Vehicle (PHEV) powertrain is the focus of this research work.

Compared to conventional optimization methods, Particle Swarm Optimization (PSO) has been proven to be an efficient and precise optimization method for nonlinear constrained optimization problems. Considering multiple nonlinear boundary conditions associated with the optimization of powertrain component sizes of a PHEV, PSO is deemed to be appropriate method to determine the optimal sizing of the PHVE powertrain components (e.g. engine power, electric motor power, battery energy capacity) for a PHEV vehicle. The cost function and boundaries are determined by the dynamic-equations that represent the performance requirements and design constraints. Upper boundaries of component sizing for engine, electric motor and the battery are used in determining the major nonlinear constraints of the optimization problem.

2. Modeling

Components of a PHEV including plenary gear, engine, and energy storage unit are modeled in this section to be used in optimization process. The power split PHEV model configuration is shown in Figure 1. In this model planetary gear set is used whose sun gear is connected to the generator and the carrier gear is connected to the engine. The output of this planetary gear set is connected to the motor through a torque coupler which gives its output to final drive and wheels.

![Figure 1: PHEV power split schematic](image)

A simplified model has been developed to formulate and solve the PHEV component optimization problem. To obtain the generator speed and torque, the planetary gear relationships are used as

\[ \omega_g = k_1 \omega_e - k_2 \omega_r \]  \hspace{1cm} (1)

\[ \tau_g = k_3 \tau_e \]  \hspace{1cm} (2)

where \( k_1, k_2, \) and \( k_3 \) are the constant values of gear ratios corresponding to the planetary gear set, and are the engine speed and engine torque respectively and \( \omega_r \) is the speed that is demanded from the ring gear. The motor torque and speed relations are according to the equations (3) and (4) mentioned above.
\[
\tau_m = \tau_r - (\beta_1 \tau_g + \beta_2 \tau_e)/\beta_3, \quad (3)
\]
\[
\omega_m = \omega_r, \quad (4)
\]

where \(\tau_g\) is generator torque and \(\beta_1, \beta_2\) and \(\beta_3\) are constants that can be derived from the dynamics of the planetary gear set [12]. Here \(\tau_r\) and \(\omega_r\) are the torque and the speed demands from the ring gear of planetary gear according to the drive cycle. They are calculated using the following equations

\[
\tau_r = \frac{e^{-at}}{\mathfrak{R}} (\tau_{req} + \tau_i), \quad (5)
\]
\[
\omega_r = \frac{\mathfrak{R}}{r} v, \quad (6)
\]

where \(\alpha\) is the time delay of the driver, \(\mathfrak{R}\) is final drive ratio, and \(v\) is the vehicle speed. \(\tau_{req}\) is calculated using a Proportional Integral (PI) controller shown in the equation (7) to model the driver response, and \(\tau_i\) is obtained as follows:

\[
\tau_{req} = K_p a + K_i v, \quad (7)
\]
\[
\tau_i = r (mg \sin(\xi)) + f_0 + f_1 v + f_2 v^2, \quad (8)
\]

where \(K_p\) and \(K_i\) are the PI controller gains, and \(a\) is the vehicle acceleration. \(\xi\) is the grade, \(m\) is vehicle mass, \(g\) is gravity and \(f_0, f_1, f_2\) are the vehicle curve fit parameters for aerodynamic drag and other losses.

The losses occurring in the motor and generator are obtained using lookup tables. The inverter loss is considered neglected.

\[
\text{Figure 2: Simplified battery model}
\]

The Thevenin model of the battery as a voltage source and internal impedance is illustrated in Figure 2. The open circuit voltage source and internal resistance are functions of State of Charge (SOC) and are calculated using lookup tables, provided by the battery manufacturer.

\[
i = \frac{V_{oc} - \sqrt{V_{oc}^2 - 4R_b P_b}}{2R_b}, \quad (9)
\]

where \(V_{oc}\) is open circuit voltage of the battery, and \(R_b\) is the internal resistance of battery.

The output voltage \(V\) of the battery is obtained from the simplified battery model as follows:

\[
V = V_{oc} - R_i i \quad (10)
\]

The State of Charge (SOC) of the battery is calculated by integrating the current over the time interval. The SOC value corresponding to the optimum set of operating points is recorded as previous SOC value for the next time interval. Below is the equation that is used to calculate SOC for each time interval:

\[
\gamma_k = \frac{1}{C_{max}} \int_{t=k-1}^{t=k} i dt + \gamma_{k-1}, \quad (11)
\]

Where \(\gamma\) is SOC, \(C_{max}\) is maximum ampere-hour capacity of the battery, and \(k\) is the time interval index.

<table>
<thead>
<tr>
<th>Table 1: Model Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Final Drive Ratio (\mathfrak{R})</td>
</tr>
<tr>
<td>PI controller gains</td>
</tr>
<tr>
<td>(K_p)</td>
</tr>
<tr>
<td>(K_i)</td>
</tr>
<tr>
<td>Driver model time delay (\alpha) (s)</td>
</tr>
<tr>
<td>Vehicle curve fit losses</td>
</tr>
<tr>
<td>(f_0)</td>
</tr>
<tr>
<td>(f_1)</td>
</tr>
<tr>
<td>(f_2)</td>
</tr>
<tr>
<td>Mass of vehicle (m) (Kg)</td>
</tr>
<tr>
<td>Radius of wheel (m)</td>
</tr>
</tbody>
</table>
3. Problem Formulation

The optimization cost function is the fuel consumption, therefore, we expect to improve the fuel economy of the vehicle by optimizing the sizes of the components. The performance of the vehicle is maintained from the optimized vehicle. General mathematical expression of the optimization problem can therefore be written as:

\[
\min_{X \in \Omega} F(X) \quad X = [P_M, P_E, NBM, FC]^T
\]

\[
s.t. \quad C_u(X) > 0 \quad u = 1, 2, 3, ..., k
\]

where:
\(\Omega\) is the solution space,
\(P_M\) is the power of the electrical motor,
\(P_E\) is the power of the engine,
\(NBM\) is the number of the battery modules,
\(FC\) is the fuel consumption, and
\(C_u\) nonlinear functions of the design constraints (performance requirements).

In addition, the vehicle performance requirements are defined in form of constraints in the optimization problem and are measured to ensure that the vehicle performance is not sacrificed during the optimization. In this study, the performance of a Toyota Prius vehicle was used. The performance requirements are:

- Acceleration of 0-60 mph 10 sec
- Maximum speed 104 mph
- Maximum speed at 60 mph in a 6% grade road.

The engine specifications are as follows:

- Inline 4-cylinder DOHC
- Displacement of 1,497 cc
- Compression Ratio of 13.0:1
- Maximum Horsepower of 51 kW at 4500 rpm
- Maximum Torque of 82 ft.-lbs.- at 4200 rpm

The battery module has the following specifications:

- Battery module contains 11 cells in parallel and 8 elements in series
- Total Number of Modules is 7
- Cell Normal Voltage is 3.3
- Cell Minimum Voltage is 2.5
- Cell Maximum Voltage is 3.6

The optimization objective function includes the power of electric motor, the number of battery modules, and the fuel consumption. All variables were normalized and weighted to form the objective function as:

\[
F(P_M, P_E, NBM, FC) = w_1 \left( \frac{P_M}{P_{M,max}} \right) + w_2 \left( \frac{P_E}{P_{E,max}} \right) + w_3 \left( \frac{NBM}{NBM_{max}} \right) + w_4 \left( \frac{FC}{FC_{max}} \right)
\]

where \(w_i\) are the weighting factors of the objective function variables.

Boundaries have been determined by the dynamic-equation representation of the performance requirements, and the design constraints. Maximum possible sizing values of three most significant components, electric motor and the battery, have been used in determining the major nonlinear constraints of the optimization problem.

To determine the engine power, the size of motor, and the number of battery modules, the extreme values are computed with the following equations:

Maximum acceleration determines the peak power of the electric motor as:

\[
P_{M,max} = \frac{1}{2\cdot t_f} m V_f^2 .
\]  \quad (13)

The minimum power of the electric motor is determined by the power required to run the vehicle at a constant speed on a road with a gradient slope as

\[
P_{M,min} = m g f v_1 \cos \alpha + m g v_1 \sin \alpha + \frac{1}{2} \rho C_d A v_1^3 .
\]  \quad (14)

The Number of Battery Modules is determined by the minimum voltage required to run the electric motor as follows:
The peak power required by the electric motor determines the maximum number of battery modules as follows:

\[
NBM_{\text{min}} = \text{Round}\left( \frac{U_{M,\text{min}}}{U_{B,\text{min}}} \right). \tag{15}
\]

The minimum power required from the engine can be calculated with mean cruise speed as follows:

\[
P_{E,\text{min}} = \frac{1}{\eta_T} \left( m g f v_1 + \frac{1}{2} \rho C_d A v_1^3 \right) \tag{17}
\]

The maximum power required from the engine can be determined by either at the maximum cruise speed or the power required on the road with a slope at a constant speed going uphill as follows:

\[
P_{E,\text{max}} = \max(P_{E,1}, P_{E,2}), \tag{18}
\]

\[
P_{E,1} = \frac{1}{\eta_T} \left( m g f v_{\text{max}} + \frac{1}{2} \rho C_d A v_{\text{max}}^3 \right), \tag{19}
\]

\[
P_{E,2} = \frac{1}{\eta_T} \left( m g f v_{\text{max}} \cos \alpha + m g v_{\text{max}} \sin \alpha + \frac{1}{2} \rho C_d A v_{\text{max}}^3 \right), \tag{20}
\]

Where

- \( v_1 \) is speed at 6% grade,
- \( v_{\text{max}} \) is maximum cruise speed,
- \( v_h \) is constant high speed,
- \( f \) is coefficient of rolling resistance,
- \( m \) is the mass of the vehicle,
- \( \alpha \) is the grade angle,
- \( A \) is the frontal area of the vehicle,
- \( C_d \) is the air drag coefficient,
- \( \rho \) is the air density,
- \( V_f \) is the acceleration speed,
- \( U_{M,\text{min}} \) is the minimum voltage of the motor,
- \( U_{B,\text{min}} \) is the minimum voltage of the battery,
- \( \eta_T \) is the powertrain efficiency,
- \( D_p \) is the specific power of the battery, and
- \( m_M \) is the mass of the battery module.

Using gradient-based algorithms, the optimized solution can be found. However, since these algorithms depend on the gradients to find the optimum solution, they do not always result in the global maximum or minimum. Therefore, derivative free algorithms such as Genetic Algorithm (GA), DIRECT, Dynamic Programming, Simulated Annealing, and Particle Swarm Optimization methods can be used to guarantee a global solution for the optimization problem.

As indicated earlier, a PSO based component optimization of a PHEV powertrain is the focus of this paper. Thus a PSO algorithm is introduced to this problem in order to find the global minimum for the defined objective function.

Particle Swarm optimization was developed by James Kennedy and Russell Eberhart in 1995 [8]. The algorithm is based on the social behavioral model of the society. The system is initialized with a population of particles with their own position and velocity values in n-dimensional space. The particles fly through the solution space by following current optimum particles using the equations defined by the PSO algorithm as follows:

\[
V(k + 1) = w \cdot V(k) + c_1 r_1 (p\text{Best}(k) - x(k)) + c_2 r_2 (g\text{Best}(k) - x(k)), \tag{21}
\]

\[
x(k + 1) = x(k) + V(k + 1), \tag{22}
\]

For the next iteration, the velocity of each particle is calculated by equation (21), and equation (22) is the position of the particle for the next iteration. Here \( c_1 \) is the cognition learning rate, \( c_2 \) is social learning rate of particle, and \( w \) is the inertial weight which enhances the performance of PSO in various applications [11]. \( r_1 \) and \( r_2 \) are random numbers between 0 and 1. pBest is the particles own best position and gBest is the global best position determined by comparing the pBest of all particles. The particles will be updated using these equations iteratively until the optimal solution is obtained or the number of iterations are exhausted.
This particular PSO technique was developed for unconstrained optimization problems. However, researchers have developed various versions of PSO algorithm, which can also be used for constrained optimization problems. In [17] Gregorio proposed a PSO approach with variation in velocity computation formula, turbulence operator and different mechanism to handle the constraints. The penalty function approach, can also be used for solving constrained optimization problems [11]. In this approach, an additional penalty function is added to the fitness function to replace the constraints.

Xiaohui and Eberhart suggested a method with some modifications in the PSO algorithm [16]. The two modifications to the original PSO algorithm are: all the particles have to be reinitialized in the feasible space, and only the feasible points are assigned for the gBest and pBest variables. Therefore, the PSO algorithm always starts and obtains values in the constrained region. Thus, the motion of the particles is always in the feasible solution space.

For this constrained optimization problem, the extremes are determined by the dynamic equations related to the performance requirements of the vehicle.

The constrained optimization PSO algorithm flow chart is shown in Figure 3.

4. Simulation Results

This constrained problem was formulated and solved using the modified PSO algorithm. The extremes of the problem and the objective function were implemented together. Therefore, for each particle in every iteration, the simulation runs in PSAT according to the parameter values that have been computed using PSO depending upon the gBest and pBest values and the velocity calculations. The optimized values obtained from PSO were used in a PSAT model, to calculate the fuel consumption, other performance parameters, and the cost function value.

We considered the existing configuration and component sizing of a Toyota Prius as initial condition. These values are listed in Table 2.

<table>
<thead>
<tr>
<th>Component</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator</td>
<td>52 kW (peak) PM Motor</td>
</tr>
<tr>
<td>Energy Storage</td>
<td>5 kWh Li Ion Battery</td>
</tr>
<tr>
<td>Motor</td>
<td>50 kW PM Motor</td>
</tr>
<tr>
<td>Gearbox</td>
<td>Planetary Gear</td>
</tr>
<tr>
<td>Engine</td>
<td>57 kW Engine</td>
</tr>
</tbody>
</table>

The component limit values are listed in Table 3. These values were calculated using initial conditions and dynamic equations of the performance requirements.
Table 3: Parameters for test functions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_M$ (kW)</td>
<td>30</td>
<td>75</td>
</tr>
<tr>
<td>$P_E$ (kW)</td>
<td>40</td>
<td>85</td>
</tr>
<tr>
<td>$NBM$</td>
<td>6</td>
<td>20</td>
</tr>
</tbody>
</table>

Since PHEVs’ major focus is urban driving, the simulations were obtained using the EPA Urban Dynamometer Driving Schedule (UDDS), for 5 times consecutively. The UDDS drive cycle is 7.45 miles in 1369 seconds. Table 4 shows the characteristics of this specific drive cycle.

Table 4: UDDS cycle characteristics

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Maximum</th>
<th>Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (mph)</td>
<td>56.7</td>
<td>19.57</td>
<td>14.69</td>
</tr>
<tr>
<td>Acceleration (m/s$^2$)</td>
<td>1.4752</td>
<td>0.505</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Figure 3: UDDS drive cycle

For the PSO algorithm, the following parameter values were used $c_1=2.6$ $c_2=1.5$ and $w=0.6$. In addition, the population size for the PSO was set to be 10, and the maximum number of iterations 30. The main reason to limit the iterations and population number was the limits in simulation time.

The optimized size of components and the original values of a Toyota Prius is tabulated in Table 5. The cost function demonstrates the fuel consumption of the vehicle and compared to the original design, a 32.26 mpg was obtained using PSO.

Table 5: Comparison of the component sizes and cost functions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PSAT default</th>
<th>Optimal</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_M$ (kW)</td>
<td>52</td>
<td>58</td>
<td>kW</td>
</tr>
<tr>
<td>$P_E$ (kW)</td>
<td>57</td>
<td>51</td>
<td>kW</td>
</tr>
<tr>
<td>$NBM$</td>
<td>7</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>Fuel cons.</td>
<td>103.52</td>
<td>134.78</td>
<td>mpg</td>
</tr>
<tr>
<td>$CO$</td>
<td>0</td>
<td>0</td>
<td>g/mile</td>
</tr>
<tr>
<td>$NO_x$</td>
<td>0</td>
<td>0</td>
<td>g/mile</td>
</tr>
<tr>
<td>$HC$</td>
<td>0</td>
<td>0</td>
<td>g/mile</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>86.2</td>
<td>74.8</td>
<td>g/mile</td>
</tr>
</tbody>
</table>

In order to validate the configuration that has been found through the optimization process, the default model and the optimized model were simulated in PSAT. The results show a significant improvement in the fuel economy of the vehicle with the components that were sized by using PSO optimization algorithm compared to the default configuration of the vehicle model. Thus, the main objective of the study, enhancing the fuel economy, has been achieved.

Since the simulation time was significantly long, the optimization problem has been restricted to some certain amount of iterations and parameter values. Therefore, for further enhancement of the system, number of iterations and number of particles that are searching for the global extremes of the scheme can be increased and thus it might be possible to further refine the configuration of the PHEV components.

5. Conclusions

The gradient free algorithm particle swarm optimization was used to determine the optimal configuration of the component sizes to achieve a better fuel economy. Therefore, a simplified model of a power split plug-in hybrid electric vehicle powertrain was developed for a plug-in hybrid electric vehicle in PSAT. This simplified model was used along with PSO algorithm to determine the optimal sizes of the major components of the vehicle such as, engine power, motor power, and battery energy capacity. These values were constrained by the performance requirements. The computed optimum component sizes were then implemented on the PSAT model. The simulation results from this new configuration were compared with those from the default PSAT model configuration.
References


Acknowledgements

We would like to acknowledge and extend our gratitude to Mr. Harpreetsingh Banvait, PhD student in Electrical and Computer Engineering, IUPUI for his help in this research work. In addition, we would like to thank Mr. Aymeric Rousseau and Mr. Phillip Sharer of Argonne National Lab, IL, USA for their help with PSAT related questions.

Authors

Emrah T. Yildiz

Mr. Yildiz is currently employed at Cummins, Inc. He received his Master’s Degree in Mechanical Engineering from IUPUI in 2010. His MS thesis focused on PHEV component size optimization of Plug-in Hybrid Electric Vehicles using Particle Swarm Optimization.
Quazi R. M. Farooqi

Mr. Farooqi is a graduate student in the department of Mechanical Engineering, IUPUI. He is pursuing his Master’s in Mechanical Engineering and working as a research assistant for PHEV research project.

Sohel Anwar

Dr. Anwar is an Associate Professor and Graduate Chair in the department of Mechanical Engineering, IUPUI. He received his Ph.D. from University of Arizona, Tucson, AZ in 1995. His research interests are Mechatronics, Intelligent systems, Hybrid Vehicle Control, and X-by wire system modeling and control.

Yaobin Chen

Dr. Chen is the Chair and a Professor in the department of Electrical and Computer Engineering, IUPUI. He received his Ph.D. in Electrical Engineering from Rensselaer Polytechnic Institute, Troy, in 1988. His research interest is in modeling, optimization, simulation, computation Intelligence and control of complex systems in various application fields.

Afshin Izadian

Dr. Izadian is an Assistant Professor in the department of Engineering Technology. He received his Ph.D. in Electrical Engineering from West Virginia University, Morgantown, WV in 2008. He has been a postdoctoral researcher at the University of California at Los Angeles (UCLA). His research interest is in Nonlinear Controls and Fault Diagnosis, and Controls of Renewable Energy Systems.